One More Song

Ambiguity and Clarity of Pitch in Barbershop Harmony

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Abstract

Barbershop harmony is a music style with focus on unified, blended chords. Relation of concurrent voices to simple harmonic ratios has been widely discussed in theory, but the pitch ambiguities of pre-arranged scores and the complexities of vocal performance create substantial challenges to the study of barbershop intonation. While mathematical tuning prescriptions do not account for the flexibility and deviation that are acceptable and perhaps even common, harmonic ratios appear to have an underlying influence on the nature of harmony.

This treatise claims that barbershop uses approximation of simple harmonic ratios within each chord in order to encourage perceptual fusion of separate voices into a greater whole. However, melodic tuning and chord-to-chord tuning are more flexible and are influenced by voice-leading and categorical conceptions. Tuning details and arranging choices are both influenced by the interaction of melody and harmony. Some situations clearly imply basic harmonic ratios, while other situations are rough and ambiguous. The contrast may be musically effective, though different arrangements vary in their amount of ambiguity. Also, absolute pitch range is important; harmonies that fit into series with low (or even infrasonic) fundamentals are rougher than the same ratios in a higher range.

This treatise also includes discussion of the basic structures of music as related to barbershop harmony and informed by concepts from music psychology. Analysis of an example piece shows how relating the score to harmonic ratios can provide valuable insights. Precisely tuned recordings are included to demonstrate the issues in the discussion. Understanding of these issues will be of value to singers, arrangers, and theorists, while also being of use in future empirical research and in developing universal theories of harmony.

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Introduction

The first barbershop harmony I ever heard was the Buffalo Bills on the soundtrack to *The Music Man.* I loved the marvelous sound but was not particularly drawn to it over other music. Years later, a college friend asked me to join his barbershop quartet. I had sung in school choirs and thought of myself as a confident sight-reader, but my barbershop baritone part seemed bewildering. Inappropriately, I treated my baritone part as a melody and sang loud and clear, even as I struggled to make sense of it. One day, an older singer at a convention said something to me like: "no, that half-step needs to be smaller than the earlier large half-step." Such an idea was completely outside of all the music instruction and experience I had up to that point — including advanced music theory courses, and I was baffled.

Eventually, I was singing with my quartet when a chord really locked and I experienced the barbershop *ring*. Describing and defining ring is not easy, but it involves a blend that is rich, resonant, and sustained with relative stability. The individual voices lose distinction, and a unified sense of a greater whole is achieved. With further practice, I became more aware of what to listen for to ring chords.

My experience of singing barbershop harmony was quite different from listening to it. In some sense, the singer transcends his ego and loses sense of self as his voice is subsumed in the quartet sound. Even secular singers often describe their barbershop experiences in somewhat spiritual terms, referencing connection to something larger than themselves. This connection is manifested in especially strong social bonds among barbershop singers. The effect is addictive, and singers at conventions are known to stay awake all night ringing chords in hotel lobbies. Many music styles exhibit a whole that is greater than the sum of the parts, but not all styles are

as predominantly focused as barbershop on coalescence into a unified single sound. The use of the a cappella voice also makes participation especially personal.

After I had my initial first-hand experience, I wanted to understand it more technically. My quartet was still struggling to achieve consistency, and I was finding much of barbershop coaching to be unclear. Some students learn well through imagery, metaphor, and other indirect teaching; while other students prefer more concrete, explicit, scientific approaches. While I appreciate and respect both approaches, I prefer the latter.

Vocal pedagogy has many inherent challenges because the voice cannot be seen, and a step-by-step mechanical guide is not as feasible as with an instrument. Singers must use indirect feedback from listening as they learn to control their voice. Furthermore, one's voice sounds different to other listeners than to oneself. Because of these and other problems, vocal coaches tend to talk vaguely and metaphorically in ways that are often open to wide-ranging interpretation. Aside from this indirect teaching, vocal music is mostly learned through rote experience. One motivation for my studies which have led to the present treatise was a desire to improve technical understanding both for my own sake and to teach others.

I did not initially intend to get greatly involved in barbershop theory. After my first inspiring barbershop experiences, I found myself dissatisfied with my other musical activities: instrumental performance (guitar and Chapman Stick primarily), composition, and recording. I wanted to get the blended, ringing sound of barbershop harmony on my guitar; but when I used my new-found pitch sensitivity to tune one chord well, other chords sounded wrong. I also found that the sounds I wanted were not readily accessed from my keyboard or from music composition software. In my search for answers, I discovered a whole world of tuning theory and

a lot of overwhelming jargon and mathematical ratios and formulas. Initially, I only wanted to find an alternative guitar fretting or keyboard tuning to use to get a consistent *ring*, but the answers weren't simple.

I undertook an exhaustive search that eventually covered dozens of books and online sources on acoustics, physiology of the ear, historical and experimental tuning and temperament, and international music styles. I asked professors at my college about my questions and was frustrated to find that only a few showed interest and they were often no more knowledgeable than I. Within the barbershop community, I found more interest and knowledge but the answers were still unclear. The greatest expertise seemed to be among an internet community of tuning theorists; although many of them seemed to be absorbed in complex mathematical charts and formulas, sometimes veering toward numerology and extreme levels of abstraction far removed from real-world music experience.

As I studied and experimented, I found that some aspects of tuning were inextricably linked to other elements of music. For example, at fast tempos, tuning details become less perceptible. To hear the subtleties of tuning, I had to play slowly and let tones waft over each other in long, sustained chords. I speculated that tuning further affects tempo because poor tuning would lead musicians to play quickly to mask the sounds of unpleasant chords, whereas more effective tuning would encourage slow tempos as musicians appreciate the sound of each chord. Timbre also has a major impact on tuning. The decay of my nylon-string Spanish guitar somewhat masks tuning problems, but the clean ring and sustain of the Chapman Stick (an amplified tap-style string instrument) makes tuning more apparent.

Today, I have concluded that there can be no perfect tuning that works for every application. No instrument will ever be able to substitute for a barbershop quartet. Furthermore, no barbershop tuning prescription can be found to be optimal for all the expressive and practical goals a quartet may have. The tuning subtleties alone are too complex to be easily programmed. Moreover, the sound alone is not the objective; it is the human experience of bringing together four voices that is most significant. Still, understanding of tuning is possible to some extent and is valuable in studying the style. In sharing the insights I have learned, I hope that singers and theorists will gain critical awareness and further appreciation of barbershop harmony, and that musicians of all sorts may benefit from the perspective that barbershop offers to music generally.

Background Overview and Discussion

In my research and personal experience, I have found a few basic conceptions that singers and theorists tend to hold regarding barbershop harmony. Participants without formal music training often emphasize implicit exposure, trial-and-error, intuition, and the vague experience of locking and ringing chords. Many singers and arrangers, however, are familiar with basic Western music theory and discuss barbershop with reference to scale degrees and popular chord names. Many coaches, directors, and singers emphasize vocal technique, particularly tone quality and match between voices. Finally, singers with backgrounds in mathematics and physics discuss barbershop with reference to the harmonic series and tuning theories. Of course there are also general issues of cultural associations, emotional expression, lyrical expression, and so on. People with any of these perspectives may treat them in more or less prescriptive or dogmatic fashions. I have studied all of these perspectives and have come to

believe that a holistic understanding of these interacting elements is necessary for adequate study of barbershop harmony.

As music does not function outside of subjective human experience, all music study should be, in a broad sense, based on psychology. Though I present a theoretical analysis here, the only important element is how the music is experienced. If a theoretical distinction has no impact on experience, it is musically irrelevant. In answering questions of experience and perceptibility, future studies could include controlled listening tests and various extensive or intensive participant and listener surveys, among many other approaches.

I hope this paper will be of interest to wide audience including music theorists, music psychologists, musicologists, music teachers, and barbershop singers and arrangers. Therefore, I have included overviews of music theory concepts, psychological issues, and barbershop basics. The following overviews, however, are not simply a review of common elementary ideas but are instead particular perspectives and clarifications that are a necessary framework to the later analysis. Ideally, readers should already be familiar with Western music notation, scale degree terms, and chord terminology. It is my hope, however, that this overview along with the pitch graph illustrations and accompanying audio will make the ideas somewhat accessible to readers without significant music theory background.

Categorical versus continuous perception and conception[1]

Some perceptions are continuous, such as physical size ranging from smaller to larger and in-between with no particular division at any point. In contrast, *categorical perception*

[1] For more on general music structural, perceptual, and conceptual topics aside from harmony, including the topics summarized here, I highly recommend *Music and Memory: An Introduction* by Bob Snyder (2000).

occurs, for example, in a continuous spectrum of visible light, where people perceive somewhat distinct bands of identifiable colors. There may be a vague in-between perception around category boundaries, but within a certain range we perceive a primary color with little distinction between subtly different variants. Some types of categories may be innate, while many are learned conceptually. Such conceptual learning may be implicit, explicit, or both. Another common example: specific categories of speech phonemes are learned depending on specific language. There is, however, a universal disposition to conceive phonemes categorically. Various stimuli may be experienced more or less categorically; in other words, there is a range between completely categorical and completely continuous perception or conception.

Pitch is generally perceived continuously, but the structure of music is built around categorical conceptions imposed over the continuous range of pitch. Western music theory describes scales and chords in clearly categorical terms. Though the Western categories do not account for much of the world's music, the presence of pitch categories in music is essentially universal. Also, just as speech phonemes vary but are restricted by the common limitations of the human voice, there are general facts about sound and human perception that greatly influence possible pitch categories in music. Significantly, a large portion of pitch categories around the world seem to relate to low-integer ratios (Stevens & Byron, ch. 2 in Hallam et al., 2009).

Category-based music theory terminology fails to account for the continuously-perceived nuances of pitch. Traditional Western theory tends to assume certain contextual tendencies for pitch categories even though continuous pitch movements are remarkably capable of overriding any such expectations. Because voices naturally glide from pitch to pitch, continuous perceptions are a factor in vocal melodies in any style (aside from perhaps yodeling and overtone singing).

Basic elements of music structure in barbershop context

Music is traditionally described as comprised of *rhythm*, *melody*, *harmony*, *timbre*, and *texture*. Though not typically considered structural components of music, two further elements of sound perception are *loudness* and *localization*. Finally, *form* is generally considered independently, though it is in some sense a large-scale aspect of rhythm. Form is only perceived in long-term memory, unlike the other parameters listed here. All these elements interact and overlap in complex ways. Additionally, there are psychological parameters such as expectation, pattern perception, association, familiarity etc. that exist both within and independently of the structural elements of music.

Loudness and localization

Loudness and localization affect overall music perception in many ways. Pitch sensitivity varies with loudness. Issues of harmonic blend vary with the relative loudness among parts. Localization can greatly affect a singer's experience when standing in various formations in a quartet. Despite their significance, loudness and localization will not be discussed further here.[2]

Rhythm

Rhythm is arguably the most fundamental element in music. Beyond simply timing, rhythm is the subjective grouping and phrasing of events over time. Stronger and weaker events are conceived (whether or not there is any objective distinction), and these are then grouped in various ways. Accents and groups are present on multiple hierarchical levels with a primary level, smaller subdivisions, and larger phrasing. Regular pulse and metered accents are common in music but are not necessary for rhythmic perception. Rhythm in music is often related to

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[2] In accompanying audio examples that are not specified otherwise, I used a subtle panning of voices near the middle of the stereo field and a generally even loudness with a slightly more prominent lead melody.

rhythm in speech, although there are also strong connections to physiology (heartbeat, breathing), movement (walking, dancing, working, etc.), and general time perception. The perception of rhythm exists mainly within short-term memory.

Rhythm in Western music notation is often vague. Symbols indicate proportional timing, but timing alone is not rhythm. The note immediately following a bar-line (i.e. the *downbeat*) indicates an accent point. Phrase markings and slurs may be used to indicate groupings, though these are often absent in published scores. Groupings and accent location from phrase to phrase, within phrases, or within beat subdivisions are often open to individual interpretation or ruled by convention. This interpretation is a large part of learning and performing music from written scores. Interpretation of rhythm is greatly affected by other musical parameters and vice versa.

In barbershop, the vast majority of pieces are built around English lyrics. Ideally, the musical rhythm and the linguistic prosody of the lyrics will match well. In an up-tune, a steady pulse is typical, and subdivisions are often sung with a swing. Barbershop ballads are often performed without a strict pulse and are open to quite varying interpretations. An effective interpretation will usually reflect the prosody of the lyrics (which is itself open to some interpretation). It is also possible to instead use a song's melody as the focus of rhythmic interpretation. The main deviation from these guidelines is a tendency in barbershop to sustain particularly interesting or enjoyable chords (Smucker, 2010).

Pitch is greatly affected by rhythmic context (and vice versa). Accented rhythmic points get more attention and have more impact in determining tonality (see later section on melody). Pitches must be sustained for subtleties of tuning to be perceptible. Short, unaccented chords can be out of tune and still have little impact on the overall experience. Long, accented chords make

tuning differences much more apparent. In a ballad without strict pulse, quartets might move quickly through chords that tune poorly and slow down on better chords. In some cases causation may go the other direction: quartets might carefully tune only sustained chords. Also, perception of rhythmic grouping is itself affected by melodic and harmonic relationships.

Timbre

Timbre, often called tone color or tone quality, refers to all the elements of a single sound other than pitch, loudness, and localization. The essential qualification is *single* sound. Qualities such as simultaneity, identical localization, consistency over repeated hearings, and harmonic blend all contribute to vibrations fusing into a single perceptual event. Varying qualities between different timbres include the spectrum of a sound (the particular combination of vibrations with various relative intensities); transient noise (at onset or otherwise, such as breathing noise or instrumental scraping/hammering noise); and variance in pitch, loudness, or spectrum over time (slower or faster attack, vibrato, sustain or lack thereof, pitch glides/drop-off, and more).

All voices exhibit some degree of vibrato and tremolo (subtle and somewhat regular fluctuations in pitch and loudness, respectively) as well as irregular flutter and other subtle inconsistencies. Furthermore, in any natural sound, spectrums change at varying degrees of overall intensity. In other words, natural sounds do not vary in loudness without additional associated timbrel changes. Therefore, a vocal recording with vibrato can be modified to maintain a steady pitch, but fluctuations in loudness and timbre will remain. Even if the recording is then compressed to have a steady loudness, the timbre will still fluctuate slightly due to spectrum changes.

Consistent timbres without vibrato or tremolo are most effective for perception of precise tuning. Ironically, fixed-pitch instruments traditionally have the most steady timbres that can highlight pitch subtleties yet being fixed-pitch denies access to the gradations of pitch necessary for a variety of precise harmonic or melodic tuning possibilities (or intentional deviations). In contrast, flexibly pitched sound sources, including the human voice, can access any particular pitch within their overall range but have complex timbres that mask subtle tuning. Today, computer-based tools can achieve both consistent timbre without vibrato and also completely variable-yet-definite pitch. However, most programs are so tedious to control that real-time music performance with completely-defined-yet-unrestricted pitch control is not possible. Technologies have improved greatly in recent years, approaching the limits of human perception, but all options still involve compromises.[3]

In barbershop, vocal timbres have changed as the style has evolved. Older recordings exhibit varying degrees of vibrato and a tone quality that might be described as nasal. Today, barbershop singers vary in nasality while most aim for minimized vibrato except as an embellishment. However, these statements are only broad generalizations. One consistent element in barbershop is timbrel match. Quartets work to match pronunciation, timing, breathiness, etc. This synchronization minimizes the distinction of individual vocal timbres.[4] Barbershop singers make efforts to maintain consistent timbre over long, sustained tones. It could be argued that the consistency of a good quartet may even be perceived as a singular sound with a *barbershop-quartet timbre*, rather than a perception of interacting separate voices.

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[3] For example, H-Pi Instruments makes keyboards with as many as 1,266 keys designed around a layout of 205 pitches-per-octave yet tunable to any pitch. The sheer number of keys, however, makes performance challenging in unique ways compared with traditional fixed- or variable-pitch instruments.

[4] For an accessible instructional overview of vocal timbre and unified blend see *Components of Vocal Blend* (Eskelin , 2005).

The lyrics in a song affect timbre because each phoneme has a distinct sound. Like many other vocal styles, barbershop singing tends to emphasize the most open vowels of each syllable. The general shape of the vocal tract creates a set of *formants* that emphasize certain absolute pitch regions. Barbershop singers are said to vary their formants on particular pitches or phonemes to emphasize blend between the voices, perhaps to especially highlight shared upper partials to create the illusion of a fifth voice, though few empirical studies of barbershop formants have been done (Richards, 2001; Kalin, 2005).

The pitch aspects of timbre include the constituent pitches of a sound's spectrum and the change in pitch over time. Vibrato, for example, obviously has an impact on tuning precision. Also, consistency of timbre helps isolate sequential sounds into a melodic pitch stream. If the lead singer's vocal timbre is distinct from the other singers but otherwise consistent, the difference assists listeners in picking out the melody from the quartet context (see *texture* below).

Most importantly, the harmonizing of concurrent sounds is related directly to the relationship of their various partials (Sethares, 2005). This means that the timbre of the voices impacts the harmony. Dull voices with few partials may provide less harmonic content to which other voices can tune, and this may make lock and blend more difficult yet also allows more tuning flexibility. Extremely rich voices with many loud partials may dominate the sound and diminish the ability of the other singers to find a place for their voices in the mix. I have heard contradictory advice from coaches on whether richer or softer timbres are preferable. Timbrel richness seems to be a very important but little-understand element in barbershop. Future studies are needed to explore this issue further.[5]

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[5] In my accompanying recording, I used a relatively moderate timbre. Perhaps as a future supplement, I could create additional versions with dulled or enhanced harmonic richness or other varying vocal timbres.

Texture

Texture refers to the perception of concurrent sound combinations in music. Music may be *monophonic* (one sound only), *biphonic* (two sounds, often a steady drone accompanying a melody) or *polyphonic* (commonly multiple active melodies at once). Additional variants include *homophony* (blended background supporting a primary melody) and *heterophony* (similar simultaneous melodies deviating slightly from one another). Music may modulate among textures, as in additive textures which introduce new parts over time.

Barbershop is said to be homophonic, and perhaps that term fits here better than in the manner it is typically used. The majority of barbershop music is also *homorhythmic* — every part has the same rhythm. Although the melody is supposedly supported by a background, it can sometimes be blended to the extent that it loses distinction. The most common distinctly polyphonic element regularly heard in barbershop is a "bum bum" sort of rhythmic part in the bass. The piece analyzed in the present study is mainly homophonic and homorhythmic.

As mentioned earlier, if a quartet were completely fused into a single sound, the perception would be timbrel rather than textural. In other words, a listener would, in principle, hear a completely fused quartet as monophonic. Though barbershop singers generally intend to fuse their sound, the complexity of real voices makes it impossible to fuse completely. The changing relationships between the parts also discourages complete fusion.

Melody

A melody is a connected stream of pitch perceived over time. In complex musical contexts, the particular way pitches are grouped into melodies may vary among listeners though some general tendencies exist. The factors that encourage perceived melodic connections include

Gestalt principles such as proximity, similarity, good continuation, and more (Stevens & Byron, ch. 2; Stainsby & Cross, ch. 5; in Hallam et al., 2009). Because these pitch events are related over time, melody is generally perceived in a rhythmic framework.[6]

Melody perception can be broken into two primary elements: *contour* and *tonality*[7] (Schmuckler, ch. 9 in Hallam et al., 2009). *Contour* refers to the general flow of increasing or decreasing vibration speed (generally described metaphorically as the raising and lowering of pitch) and is perceived continuously. Direction of contour is readily perceptible as is differentiation between small steps and large leaps. Being continuously perceived, however, contour offers no clear distinction between similar intervals. *Tonality* refers to a hierarchical categorization of pitch regions, usually with a primary tonal center, certain secondary pitches, sometimes certain less-prominent accessory pitches, and usually with some pitch regions excluded from the system. Tonal systems are learned through both implicit and explicit experience. Tonal categories categories may be flexible and broad or specifically defined; and particular categories vary by culture, by individual listener (Hahn & Vitouch, 2002), and between — and even within — different pieces of music.

The perception and function of melodic tuning is related to the interaction between contour and tonality. Contour alone has only very basic impact on melodic tension and resolution: there is, for example, a widespread tendency for phrases to build tension with rising pitch and resolve with a descent. Tonality alone can define tension and resolution only by the degree to which a category is considered resolved. Combining the two factors, pitches in less-

[6] Melody and rhythm are largely processed independently in the brain, so it is possible that someone with injured rhythm processing could still process melodic grouping. Nevertheless, typical listeners perceive rhythmic groupings and accent simultaneously and holistically with melody; and melody and rhythm interact and affect one another.

[7] Here *tonal* and *tonality* refer to hierarchical pitch systems in a general sense not meant to refer to any specific cultural or historical practice such as Western classical *functional tonality*.

resolved tonal categories tend to resolve by smooth contour toward the nearest more resolved categories, a phenomenon known as *voice-leading.*[8] In context of smooth voice-leading that moves from one tonal category to an adjacent category, it is common and effective for the initial pitch to tune near the category toward which it leads (Eskelin, 2005; Devany & Ellis, 2008). In a harmonic context (discussed in the next section), however, quality of blend interacts with melodic voice-leading to further impact tuning precision in complex ways.

Melodic style in barbershop seems to originate from a mix of cultural traditions with a focus on popular American song from the turn of the 19th to the 20th centuries. Typically, a primary melody with a moderate pitch-range is sung by a lead singer. The tonality of the melody is very clear, with the majority of songs based in the traditional Western major mode, although some prominent harmonic sevenths are also found (see discussion later regarding harmony). Chromatic passing and neighboring embellishments are also common (although these are usually harmonized by supporting parts rather than left as truly non-harmonic). Leaps occur mostly between chord-tones (an indication of the influence of harmony on melodic structure).

Unlike some other vocal styles, the other three harmony parts are generally not intended to be heard melodically. While there are notable exceptions when harmony parts sing distinct melodic bits, the main purpose of the supporting voices is to provide harmonic context. All of the supporting parts regularly include melodic structures that would never be heard in a song's melody and are instead chosen for their harmonic functions. In particular, the baritone part can approach melodic incomprehensibility for typical Western listeners. Only by hearing (or, for experienced singers, imagining) the place of the voice in harmonic context will such a part be

[8] Common syntax of voice-leading is traditionally taught within the study harmony, but the nature of voice-leading is clearly melodic. Harmonic context can assist in defining tonal categories — thus affecting melody, but in this treatise I attempt to define and understand harmony independently from these melodic elements.

accessible. Still, the tendency of all voices to favor small pitch movements and to glide smoothly from pitch to pitch brings out some perception of melodic contour and distinct streaming in each part. On the other hand, because quartets work to make their vocal timbres and pronunciations indistinguishable, a listener's ability to perceive notes as melodically distinct is minimized. The quartet is often heard as a unified harmonic sound to such an extent that it risks losing melodic perceptibility of even the lead voice.

Analysis of melody in barbershop must be done with consideration of the categorical conceptions (not necessarily explicit) of singers and listeners. Pitches need to be adequately within particular tonal categories, particularly those in the main melody. Within a category, precise tuning is a compromise between maximizing harmonic blend and producing singable contour and effective voice-leading. The flexibility of categories may vary from person to person and even performance to performance. In general, more varied tuning encourages wider and more flexible subjective tonal categories. With fixed-pitch instruments, precise pitches are consistently reinforced and slight deviations may be heard as category-violations. In contrast, gliding voices with vibrato and inconsistent tuning will lead to tonal categories that are flexible and tolerant of small variations. All of these considerations will vary further with the training, enculturation, and attention focus of each individual singer or listener.

Harmony

Harmony involves interaction between concurrent pitches. Thus, there is no harmony if a listener perceives simultaneous vibrations as completely fused into a singular sound. In contrast, a polyphonic conception supposes that listeners hear separate tones and distinct melodic streams with focus on independent melodic functions rather than on harmonic relationships. Harmony

must obviously exist along a continuum between these extremes. Harmony could be defined as the interaction of concurrent pitches that are not fused into a completely unified timbre nor perceived as entirely distinct. The following paragraphs discuss a number of issues involved in trying to define and study harmony.

A certain pitch combination might fuse when heard in a consistent relationship. The same combination may be perceived with distinct elements, however, in a context of changing relationships that establish independent pitch streams. Though there are rare moments in barbershop where all four voices move in parallel, the typically contrary and oblique movements between the voices maintain some part distinction.[9]

Roughness and tuning

When simultaneous steady tones are near in pitch, beats (regular amplitude fluctuation in the combined waveform) occur. Beats are a factor in the more complex phenomenon of *sensory roughness,* which is perceived when simultaneous pitches are within a *critical band.* Even when fundamentals of concurrent tones are far apart, upper partials may be within critical bands and cause roughness. Interacting sounds also cause difference tones (created by intermodulation distortion in a non-linear medium, such as the inner ear); and when these difference tones are within critical bands of one another or of other partials, they can create further roughness.

Among any complex tones heard simultaneously, the situation least susceptible to roughness is that of tones with harmonic spectra tuned to low-integer ratios. Tuning to these

[9] Maintenance of part-distinction is the reason for the avoidance of parallel fourths, fifths, and octaves in classical counterpoint where listeners are supposed to follow the separate melodic streams. The following intervals have the lowest integer ratios: fourths are 3:4, fifths are 2:3, and octaves are 1:2. They blend so well that they encourage perceptual fusion. The other intervals in the Western diatonic scales are higher-number ratios and don't blend as strongly. Furthermore, those intervals (seconds, thirds, sixths, and sevenths) vary in size between major and minor versions within diatonic scales, thus avoiding fusion when heard in diatonic parallel harmonies. In barbershop, where fusion of parts is generally desirable, parallel fifths are common (parallel octaves are rare because most chords contain four distinct pitch classes).

integer ratios is called *Just Intonation.* Because real-world singers can never be perfectly precise, some minor level of roughness in all vocal harmonies is essentially guaranteed. Near Just Intonation, however, roughness is mild, and sounds can approach fusion. An individual pitch tuning that might otherwise be heard as outside an expected tonal category can be made acceptable when justified by strong blend in harmonic context. Interestingly, strong roughness masks pitch clarity and so discourages perceptual fusion while simultaneously decreasing a listener's ability to parse the music into clear, distinct streams. Thus, strong roughness harms both melodic and harmonic perception.[10]

Psychological significance of blend

The subjective sense of distinct sounds blending together might be a prominent element in the experience of harmony. Pattern perception and other aesthetic issues certainly come into play as listeners perceive coordination among otherwise separate elements. The impact of such blending may be heightened when there is *agency* conceived in the source. Both outside observers and active participants have powerful reactions to witnessing people acting or sounding as a unified group. A common speculation is that such group solidarity and coordination had a role in human evolution and the associated development of music (Cross, ch. 1 in Hallam et al., 2009). In this regard, harmony in music truly matches the metaphoric use of the term, i.e. harmony between people. Indeed, this sentiment is widespread and profound in barbershop where there is emphasis on camaraderie and social unity.

[10] Perhaps this effect explains why the same harmonies that are considered dissonant when played as block chords can be appealing when arpeggiated, as commonly done in instrumental music. Arpeggiation brings out single notes distinctly, allowing clarity of melodic perception despite harmonic roughness. Perhaps the experience of dissonance is not only due to roughness itself but also to a listener's frustration of being unable to perceive clear pitches. Consistently rough music frustrates a listener's ability to make sense of the melodies and patterns.

Chordal temporary tonality

Tonality exists both in melody and harmony. Traditional discussions about harmony often refer to chord progressions. While chord progressions are certainly not universal in music, they do have prominence in a wide variety of global styles. The implication is that a certain set of pitch classes are prominent during the time of a certain chord. Then a different (but often related) set of pitches takes over upon arrival at a new chord. This categorical prominence is similar to the way tonality is exhibited in melody alone. Just as melodies create general pitch class hierarchies around a tonal center, chords do the same around their roots. Perhaps both melodic tonality and momentary chordal tonality (which certainly impact one another) are based on similar psychological mechanisms. The relation of momentary chordal tonality to a song's overall tonality may be part of experiencing tension and release in a chord progression. Common stylistic chord progressions are essentially culturally-learned syntax built upon these more general elements of tonal relations, varying roughness, and melodic voice-leading.

Harmonic identity and implied harmony

Some advocates of Just Intonation promote the concept of harmonic identities based on ratio numbers. They describe each tone in a harmonic series as having significance based on its mathematical ratio to other pitches or to a tonal center. For example, the upper of two pitches at a 4:5 ratio and the lower pitch of a 5:6 pair would both be identified as having a quality of *5-ness*. Just Intonation theorists often suggest that non-prime numbers have a quality related to that of the number's prime factors. For example, the seventh harmonic has a prime-identity of 7, while the ninth harmonic has a prime-identity of 3 and the fifteenth harmonic has prime-identities of both 3 and 5. In most music, multiples of 2 are considered to be the same pitch class, a

phenomenon known as *octave-equivalency* in Western terminology. Because octave-equivalency is usually upheld in the harmonic-identity thesis, odd-identity is considered the next level of significance after prime-identity.

Regardless of the questionable numerology of this harmonic-identity thesis, it is possible that people can learn to conceive of a single melodic tone as having a quality associated with its role in a harmonic context (apparent or imagined, as in what is traditionally called *implied harmony*). I doubt that this is necessarily related to mathematical ratios, but seems instead to be a learned categorical conception similar to melodic tonality. Those familiar with a musical style may learn implicitly to expect certain harmonic contexts for melodic pitches and be able to imbue a single pitch with a categorically-imagined context. One may become familiar with the harmonic quality of a major third, for example, and then somehow apply that quality to their hearing of a single tone. This relation suggests a conception of some harmonic quality distinct from overall song tonality. If this harmonic quality can be sensed, then perhaps Just Intonation theorists develop implicit sensitivity to numerical-identity categories through overlearning in the course of their studies; thus their theory becomes a self-fulfilling prophecy. Of course, the natural occurrence of the harmonic series may lead to implicit recognition of harmonic ratios even in untrained listeners. Perhaps we even have some innate sensitivity to harmonic ratios.

These ideas are far from well-understood, but studies of the hypotheses seem reasonably possible. Novel chords outside of common tuning systems could be repeated and reinforced to various listeners in different contexts and then similarity or expectation tests could be administered. Brain scans could be administered while a subject hears a full chord compared to hearing single pitch while imagining its harmonic context. On a related note, Dr. Jim Richards,

author of *The Physics of Barbershop Sound* (2001), told me (personal communication, 2003) of a casual study in which lead singers performed first solo with no instruction and then again after being told to imagine the accompanying harmony. When imagining the harmony parts, singers supposedly modified their tuning to fit the imagined harmony.

Overview of barbershop style

Barbershop harmony originally developed in the late 19th Century. The influence of both European and African musical elements is certain (Averill, 2003). It is clear that the development of the style included both written arrangements as well as harmonizing by ear — known in the barbershop world as *woodshedding*. Barbershop is today defined as having the following elements: four singers (traditionally male), primarily homorhythmic structure, modest pitch range, a primary melody in a middle part, and a focus on blending consonant chords with an emphasis on chords using the harmonic seventh. The melodies of barbershop songs are rooted in the Western diatonic system, and the harmonies often follow dominant-based circle-of-fifths progressions common to Western music.[11]

When the style first arose, popular songs were mainly either folk melodies or written by Tin Pan Alley songwriters whose main business was selling sheet music. Being intended for amateur singers, both of these song sources favored melodies with small to moderate range, consistent tonality, and generally simple contour. These characteristics also happen to make the songs especially well-suited for close harmony.

[11] For basic understanding of resolution by descending perfect fifths, often taught through a "circle-of-fifths" diagram, any introduction to Western music theory will discuss this. For an introduction specific to barbershop see *Theory of Barbershop Harmony* (Szabo, 1976). Throughout the present treatise, the term *circle progression* refers to such syntactic descending-perfect-fifth chord patterns.

The four parts in barbershop are called *tenor*, *lead*, *baritone*, and *bass*. The lead sings the melody while the bass provides a strong harmonic foundation (usually the root or fifth of each chord). The tenor harmonizes above the lead's melody, and the baritone may fluctuate above and below the lead melody, fitting in whatever is needed to complete each chord.[12]

Written arrangements may be transcriptions of woodshedded pieces or may be created by trained arrangers using the piano (with its fixed, tempered pitches) and/or Western theory (with its general categories and lack of tuning specificity). I have heard anecdotally that good woodshedders may create arrangements that avoid some of the complex tuning problems often found in written arrangements. Understandably, when woodshedders struggle they seek out different pitches. Singers using a score, however, are more likely to practice the written notes without considering alternatives. I personally know singers who regularly discover alternative tones they find harmonically appealing and insist on singing those over notes in a written score.

Understanding of the issues of tuning and temperament is more widespread within the barbershop community than among musicians in many other musical styles. Modern barbershop teaching, however, largely uses traditional Western chord names and notation; and this method fails to clarify issues of tuning and other related subtleties. The other prominent learning tool is recordings that demonstrate each voice part. These recordings may or may not be representative of real-world performance and are influenced by the particular intentions and capacity of the creator. Some recordings may be unedited, while others are computer-generated or edited, such as those accompanying this treatise. Such rote training provides only implicit instruction on tuning and other issues.

[12] The best source for detailed concepts within barbershop arranging, including many referenced throughout this paper, is the *Barbershop Arranging Manual* (1980).

Just Intonation ratios and general issues of barbershop tuning

Singing in octaves[13] is likely as old as singing itself. An octave is a pitch interval with the ratio 1:2. This relationship has arguably universal significance in music. While complete explanation of octave perception is still the subject of ongoing research and debate, some general points are clear. The combined waveform of two pitches at an octave has the same period as the lower pitch alone. Furthermore, a harmonic series built on a lower pitch contains all of the notes present in a pitch one octave higher. There is inherent ease of harmonizing at the octave between men and women (or children). However, young beginning singers may sustain other simple harmonies, most often approximating *perfect fifths* (2:3 frequency ratio) and sometimes *perfect fourths* (3:4) or *major thirds* (4:5) when simply trying to match a pitch or sing at an octave.[14] Thus, simple harmonic ratios arise naturally, with prominence on lower-integer ratios.

These ratios harmonize well because the human voice has a spectrum of partials that creates a harmonic series. This means that all the upper partials[15] are exact multiples of the fundamental pitch, e.g. 100Hz-200Hz-300Hz-400Hz and so on. Note that pitch is perceived on a logarithmic scale, which means that, for example, the doubling from 100Hz to 200Hz is heard as the same interval as that of 400Hz to 800Hz. Thus, the harmonic series is heard as having intervals that get progressively smaller and smaller, as illustrated in Figure 1.

[13] So-called because the Western diatonic scale has this interval at the eighth pitch.

[14] Based partly on observations in my private music teaching.

[15] Note that, in the context of a harmonic series, the term *harmonic* may be used interchangeably with *partial*. The common term *overtone* is a poor choice that differs by one number because it excludes the fundamental. *Overtone* originates from a mis-translation of Helmholtz' German term *Oberpartialtöne*. See translator Alexander Ellis' note pp. 24-25 in Helmholtz (1885).

Figure 1. Illustration of logarithmic perception of the harmonic series.

In order to understand ratio theory, a brief overview is needed. All steady pitched sounds with continuously added energy (bowed or blown) create perfectly harmonic partials. As mentioned earlier, having harmonic partials means that all the concurrent pitches in a sound's spectrum are multiples of a starting fundamental frequency. For example, a singer or harmonic instrument sounding a fundamental of 200 cycles per second will have upper partials of 400, 600, 800, 1000, etc. The fundamental partial is the first harmonic, and the doubling is the second harmonic, etc. Thus, the interval between any two harmonics will be identical to the ratio of the two numbers, i.e. the interval from harmonic six to harmonic seven is a 6:7 frequency ratio.

When multiple complex tones (each with their own harmonic spectrum) are harmonized, their relationships may be described by frequency ratios. The combination may then be conceived as partials of a single, larger harmonic series. For example, three tones with fundamentals of 440, 550, and 660, have a combined ratio of 4:5:6, and create a complex combination that includes only frequencies that are present in a harmonic series starting at a 110. Notably, *combination tones* actually generate the missing fundamental and other partials of the larger series when harmonized tones interact.

In addition to ratios, the absolute pitch of a chord is relevant. The speed of beats created by interacting tones is the difference between their absolute frequency (for example, interacting pitches at 440Hz and 445Hz create an interference pattern of 5Hz). Additionally, the human auditory system has different sensitivity with much wider critical bands in lower frequency

ranges. Generally, ratios that create a fundamental near or below infrasonic ranges are much rougher than those with higher fundamentals. A chord with tones of 440Hz, 550Hz, and 660Hz sounds blended and smooth, while a chord with tones of 44Hz, 55Hz, and 66Hz is much rougher.

As the aim of barbershop is usually to hear a unified sound, harmonies that fit well into a single harmonic series are preferred (though this may or may not be explicitly understood by particular singers or arrangers). For example, the majority of barbershop songs end with a major chord with the structure (from bass to tenor) of root, fifth, root (octave), third — instead of the classically common order of root, third, fifth, root (octave).[16] When tuned harmonically, the ratios of the barbershop final chord are 2:3:4:5, whereas the classical closed-position chord would give the ratios 4:5:6:8. The 2:3:4:5 barbershop version also keeps the root in the lead voice below the tenor. When the higher-ratio close-position version is used, as in the final chord in the song analyzed here, it is usually in a relatively high absolute pitch range (for male barbershop harmony). The tendency of barbershop to gravitate toward low-number harmonic ratios is also shown in the the fact that the bass singer is almost always on roots and fifths of the chord, which will be usually a 2, 3, or 4 identity in the chord's combined ratio. The bass is much more rarely on the third or seventh of a chord, which would result in notably higher numbers for the combined chord ratio.

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[16] Terms such as *root*, *third*, and *fifth* refer to the position of notes in Western diatonic scale order. While the terminology is unfortunately confusing compared with mathematical ratios (for example, the *third* of the scale corresponds to the *fifth* harmonic), these cultural terms are widespread and so are the most practical choice here and will be used as necessary throughout the analysis. See Appendix D for a chart translating these terms. Any traditional music theory introductory text will also discuss these scale terms (though typically not in relation to harmonics).

Barbershop harmony emphasizes the use of harmonic sevenths, which (in strict Just Intonation) are 31 cents[17] flatter than the closest interval in Equal Temperament,[18] a noticeable difference. The harmonic seventh harmony is so important to the style that it is referred to as the *barbershop seventh,* and dominant seventh chords tuned to approximate it are called *barbershop seventh chords*. Singers do not sing such harmonies precisely to the cent but have been shown to approach this tuning, commonly singing sevenths flatter than tempered.[19]

While barbershop style is said to favor harmonic tuning within each chord[20], there are complex questions about the relative tuning from chord to chord. Common teaching says that the melody should be tuned to whatever pitch that the lead singer feels is right and that the harmony singers should adjust to blend with the lead. Alternatively, another proposal is that the bass, singing roots or fifths, should maintain connection to the tonal center and provide the tuning focus. In reality, how tuning is handled may vary among quartets, songs, sections in songs, or even particular performances. Most singers believe that maintaining an absolute tonal center throughout a song is important, but others are less concerned. Tonal centers often flatten, perhaps due to the natural tendency of the voice to relax and then drop in pitch or perhaps due to harmonic progressions that inherently flatten (as discussed later in the analysis). Experienced quartets sometimes sharpen the tonal center over the course of a song, perhaps due to excitement

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[17] A *cent* is defined as a hundredth of an equal-tempered half-step.

[18] Any division of a pitch range into equal steps is an Equal Temperament. Tuning theorists commonly discuss many forms of temperaments of many different numbers and sometimes dividing base intervals other than the octave. Here the term Equal Temperament refers to the common practice of dividing the octave into twelve equal semitones.

[19] Only very limited studies have been published, however, such as Hagerman & Sundberg (1980). However, I have seen and participated in a number of unpublished, informal studies of intonation in barbershop recordings; and although intonation tendencies seem to vary by quartet and time-period, there are examples of quartets tuning close to pure ratios.

[20] Averill (2003) claims that explicit reference to Just Intonation harmonies in barbershop originates within the barbershop revival movement in the mid-20th-century and was not necessarily present in the style's beginnings.

or to over-compensation when avoiding flatting. Regardless of measurable pitch changes, sense of tonal center is subjective. The important issue is whether singers and listeners experience the music as fitting their conceptual tonal categories. If a resolve to tonal center is experienced as fully resolved, then it is fully resolved. The range of tolerance for these categories among various people is of major importance here and calls for further study. Designing controlled studies, however, is challenging because tonal experiences are highly influenced by contexts of all sorts.

The harmonies in barbershop are too complex to be adequately realized with a small set of fixed pitches. In practice, barbershop tuning is an adaptive process influenced by a number of factors including: maintaining an accessible melody; resolving to an acceptable tonal center; and tuning individual chords to blend well, particularly on sustained and structurally important chords. As will be shown in the analysis, there are sometimes direct conflicts between these various pressures. Of course, factors such as vibrato and tempo can mask perception of subtle tuning. Furthermore, like any diverse music style, opinions vary as to what is “good” barbershop.

Analysis of *One More Song* by Joe Liles

While most barbershop arrangements are adapted from popular songs not originally intended for the style, *One More Song* (Liles, 1985) was written by a barbershop arranger specifically for barbershop singers. The song was originally published in a collection of “Barbershop Potpourri” along with titles such as *It's Barbershop Showtime*, *Let's Sing A Little Barbershop*, and *Welcome Song*, along with a number of short traditional songs. Today, *One More Song* is featured among other selections made free to the public at the Barbershop Harmony Society’s website. I selected *One More Song* for analysis because of its free

availability, its short length, and because it contains substantial analytical challenges. In contrast with more basic arrangements, *One More Song* shows the type of complexity that is common in longer, more advanced barbershop arrangements, yet it remains reasonably singable and accessible. Though this analysis contains only a portion of the various situations found in barbershop tuning, it is varied enough to demonstrate the nature of the issues.

Analysis method

Essentially, pitch in barbershop harmony can be described thus: the melody fits within certain tonal pitch categories in relation to an overall key while the other three voices harmonize in a manner that places the melody note within a certain categorical relation to a larger momentary harmonic series. For example, the lead might sing the note C, while the other parts are arranged around it to construct a harmonic series in which C is the 5th partial (this would be the tonic chord in the key of A♭). Or they could construct a different series in which C is at the 6th partial (as in an F7 chord). The tuning of the lead or the tuning of the harmony parts is a matter of the many factors discussed already such as categorical conception of tonality, sensory roughness, and surrounding melodic context.

In the analysis here, I have chosen to use interval ratios to specify tuning and discuss chord identities. Although precise ratios are used in the recorded examples, I am not suggesting that these exact ratios represent the reality of live barbershop harmony. Live singers will never be absolutely exact in their tuning. I believe instead that the harmony approximates, and is influenced by, simple harmonic ratios, gravitating toward them with varying degrees of precision. The ratios therefore represent a general categorical concept implied in each chord. Tuning to the ratios is ideal for harmonic fusion, but not necessarily for melodic voice-leading.

Essentially, harmonic ratio analysis of a barbershop score involves identifying potential harmonic tunings and then considering issues of melodic contour and overall tonality. Finding potential ratios involves comparing the structure of the written arrangement to the harmonic series to discover the best fit for each chord. Some chords are readily defined, having only one simple harmonic ratio that could reasonably fit the note arrangement. Other chords are ambiguous with more than one reasonable harmonic ratio that could fit or no readily apparent fit whatsoever. In ambiguous cases, surrounding context may clarify a preferred tuning. With wide vibrato or other pitch fluctuation or deviation, however, a chord may remain ambiguous even if a preferred tuning analysis is possible. Ambiguity can create functional tension or may allow for alternative listener interpretations which might change between approach and departure from the chord in question. Even ambiguous chords may be related to potential harmonic ratios in the minds of listeners and singers, whether implicitly or explicitly.

In some cases, melodic functions override harmony, and the function of a particular chord may not be related to blending into a simple ratio. In other cases, there may still be clear ratios between two or three of the parts but the combined ratio is less simple. In some cases, melodic context can make one harmonic tuning preferable over another. For live quartets, aside from choosing one clear solution over another (consciously or not), ambiguous situations might also be passed through roughly, simply ignoring the details and allowing some tension as they move on to more clear chords. Attention may be placed on melodic motion, pitch glides, lyric pronunciation and delivery, or other factors that do not relate to harmonic tuning. Though all these factors are undoubtedly involved, determining the particular ways in which quartets and listeners balance these issues will require further studies.

Tuning of melodic relationships from chord to chord is more flexible than harmonic tuning. While melodic tuning issues from chord to chord are discussed, the analysis emphasizes the internal structure of each chord instead of the sequential tuning of voice parts. At times, melodic tuning may require adjustment to fit a precise theoretical harmonic tuning, but as long as tuning is within a general tonal category the melodic result will be acceptable to most listeners. Whether and when such adjustments are made by live quartets is a question for further study. A prominent steady tone will draw attention to itself if it is suddenly altered in tuning, and the awkwardness of such sudden alteration is often cited among objections to Just Intonation theory. Yet with a complex human voice gliding between notes along with moderate vibrato and fluctuating timbre, sensitivity to subtle melodic shifts drops dramatically. When there is a significantly long rest or unrelated interim notes, subtle tuning changes will only be noticeable to listeners with both a strong sense of absolute pitch and active attention to tuning. If a pitch is swapped between voice parts, changes octaves, smoothly glides from tone to tone, or is in a less noticed harmony part (particularly the baritone), small melodic tuning shifts can be completely masked. Finally, the effect of harmonic blend helps distract from melodic focus on any small shift. Live barbershop, therefore, admits small melodic tuning shifts (though if they are not used, the resulting harmonic deviation may also be admissible). *Readers should keep in mind that the ratios in the analysis are meant to be general categories — not pedantic prescriptions for tuning.*

Discussion of accompanying recordings

The recordings accompanying this score analysis are edited variations of my own performance. I sang all four parts, overdubbing each. This process was used for practical reasons based on constraints of time and resources. Fully separated parts were necessary in order to

control tuning of each voice while demonstrating the concepts shown in the analysis. Though there are issues with this choice, not the least of which are the limits of my talents as a singer, there would be complex concerns with any single recording from any source.[21]

Using Melodyne from Celemony Software, I adjusted precise pitch, vibrato level, and tempo to highlight harmonic issues discussed in the analysis.[22] I have included unedited, moderately adjusted, and fully adjusted examples so that listeners have a sense of the significance of tuning precision. Though deviation from pure ratios can be effective for many different reasons, the recordings and analyses here focus on harmonic ratios in order to demonstrate their effectiveness and to propose the ratios as a harmonic basis from which deviations are made. Appendix A lists and describes the audio files, which will also be referenced in the the analysis discussion.

Critics may question the relevance of computer-adjusted recordings to the study of an art form done by live singers. It is noteworthy, however, that there exists today a widespread phenomenon of young barbershop singers creating overdubbed and edited recordings shared on YouTube. These recordings may feature a single singer on all four parts or may be collaborations including singers thousands of miles apart in different countries. A large portion of these

[21] Judging the style by analysis of only one or two quartets, as in Hagerman & Sundberg (1980) and Kalin (2005), is highly problematic. For this reason, I am using my recording to present conceptual ideas rather than analysis of live performances. Though study of live quartets, instrumental versions, and controlled computer-generated versions may be valuable, the number of parameters to address is overwhelming. For some idea of the complex issues, consider that Virtual Singer from Myriad Software uses independent recordings for each phoneme to define a given voice model and then offers the following adjustable parameters: multiple formant ratios, time ratio between open and closed vocal folds, saturation, random deviations in timing, change in formants over time, volume envelope, pitch attack (approaching each note from below), random jitter (flutter) in pitch, regular vibrato (with adjustable onset, speed, and depth), portamento between notes, and more. Live singers may, of course, change any of these and many other parameters at different parts of their voice range, or at different volumes, and randomly or expressively throughout a performance. Furthermore, any individual quartet will have various biases greatly influenced by enculturation both within barbershop and in the larger music culture and through explicit instruction they receive. As I have repeatedly emphasized, this complexity should be kept in mind when considering the significance of the present pitch-focused analysis.

[22] For further description of the process of vocal retuning with Melodyne, see Wild & Schubert (2008).

recordings are edited similarly to the examples included here. In some sense, my recording is simply an example of this new high-tech barbershop culture. Also in defending my choice for audio examples and my precise analysis, I offer the following analogy:

In the visual arts, certain geometric forms, such as the perfect circle, are significant. Few artists are able to create even near-perfect circles freehand, and perhaps certain tendencies in the form of freehand circles could be found, but this is not an indication that the concept of a circle is not their intent. Of course, art with only perfect circles is not very appealing. Astronomy was hampered for years because of blind faith in the perfect circle. Deviation from the perfect circle, whether intentional or not, is part of the beauty and interest of art and science; and yet we must, of course, understand the concept of the circle in order to recognize deviance from it. I do not want to suggest that perfect harmonic tuning is either superior to or descriptive of real singers. It seems to me that harmonic tuning represents one conceptual framework from which music may be built (and deviated from) even if it is normally as imperfect as a freehand drawing of a circle. It would be foolish to suggest that artists should avoid being conceptually familiar with the perfect circle. Likewise, it is reasonable to propose that barbershop singers (and all musicians actually) would benefit from familiarity with the harmonic series and the sound of precise harmonic blend. Furthermore, just as the compass is a valid tool in drafting as well as in visual art, precise tuning with computer technology is now a valid tool in the ongoing evolution of music. Just as we certainly cannot extricate the influence of the piano with its tempered tuning from the broad context in which barbershop harmony evolved, we must also accept the inevitable influence of computers today. And while my accompanying recordings are not meant to be representative of live quartets, they do demonstrate some of the issues that quartets face.

Regarding elements such as rhythm, pronunciation, dynamics, vocal quality, and more, my recordings attempt to deliver interpretation that does not drastically differ from the score but is somewhat representative of the sorts of stylistic interpretation typical of barbershop ballads. It is influenced in part by the way I have heard and sung this particular song in my barbershop experiences. Parameters of vocal technique, pronunciation and speed of phoneme transition, vibrato, dynamics, etc. all influence the musical result in numerous ways. I attempted to sing in a relatively neutral fashion with natural inflection and even tone.

Discussion of score analysis and pitch graph figures

To relate to the language both theorists and barbershoppers typically understand, I have chosen to do a modified macro analysis. Alongside the score are chords by letter name along with Roman numerals for scale degrees of roots with uppercase numerals for major triads and lowercase for minor (and diminished) triads. Arabic numerals indicate chord extensions such as sevenths. Slurs show circle progressions, and dotted slurs show alternative progressions with similar strong cadential leading. Classically-oriented theorists should note that the analysis does not indicate figured-bass style inversions; and secondary-dominant chords are indicated by scale degree (e.g. II7). The complete analyzed score is in Appendix B along with further discussion.

To facilitate more accessible understanding of pitch and harmony, I have created a graph that illustrates the pitch of each part and how the four parts form partials of a larger harmonic series. Numerals under each pitch on the graph indicate the analyzed potential harmonic ratios for each chord. The full graph is in Appendix C along with further explanation. Side-by-side excerpts from the score and the graph will be shown throughout the following discussion.

Measure 1: Voice-leading, tuning the 7th of II7, and a passing 6th

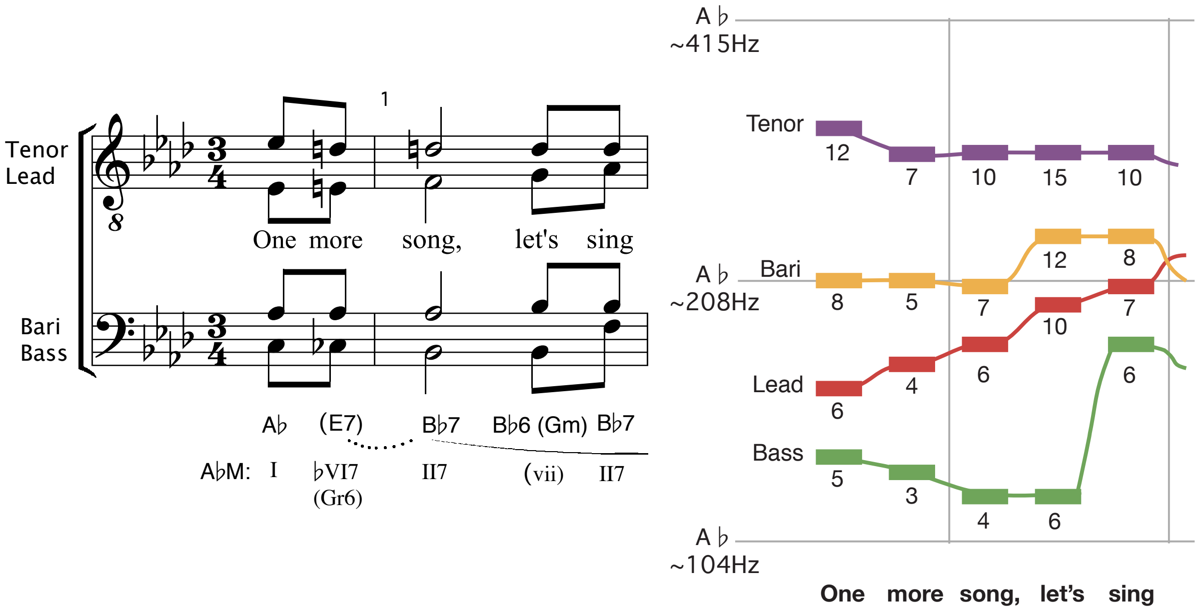

Figure 2. An inverted tonic chord pickup leads into a II7 chord on the downbeat of measure 1.

The initial chord with doubled fifth and the bass on the third is atypical for barbershop. This inversion results in a combined ratio with higher integers than if the bass were on the root or fifth of the chord. As written, the chord provides more tension on the pickup and very smooth voice-leading into the downbeat. Were a different chord arrangement used, it might call for changes in the following chords in order to maintain effective, singable voice-leading. This is an example of melodic voice-leading getting preference over blended harmony. The tension helps confirm the rhythm as an end-accented grouping with focus on the II7 chord and the word "song." Thus harmonic structure interacts with melody, rhythm, and lyrical meaning. The inverted chord is a little harder for quartets to lock and is not great for humming a preparation chord, but it goes by quickly, so the exact tuning and blend will not be the focus.

The spelling of the second chord ("more") is confusing. In fact, this chord is spelled differently in the two publications I have seen. The spelling that is used in the newest publication (and in this paper) is based on chromatic voice-leading. Regardless of spelling, the only sensible tuning is essentially barbershop seventh chord with the lead on the root. Classical theorists might call it a *German augmented sixth chord* due to its particular voice-leading. Jazz theorists might think of it as some variant of *tritone substitution*. If considered independently, the tuning of the lead would not need to be precise because it is simply a chromatic melodic motion with contour-based rather than tonal function. The bass's contrary motion is likewise mainly a contour function. Some absolute tuning decision is necessary when creating precise recordings such as the audio examples included here; and because the baritone started the song on the tonic and is sustaining the pitch, it seems reasonable for the chord to be tuned to the baritone. Most quartets, however, would not invest time in the tuning of this chord. Instead, it will be treated melodically, along with the first chord, as a quick tension leading into the downbeat. Despite the melodic voice-leading focus, it is notable that even quick passing chords like this are nearly always structured to be one of the main accepted chord types in barbershop.

The downbeat on "song" in m. 1 is the first strong, sustained harmony. Here we encounter the first tuning challenge. The best harmonic ratio on "more" gives a 5:7 between the baritone's third and tenor's seventh; but the same written notes on "song" are reversed in their chord identity, becoming an octave-inverted 7:10, as illustrated in Figure 2. In tempered tuning, there is no distinction, but the mathematically precise 5:7 ratio is 583 cents, while 7:10 is 617 cents (the difference between the 5:7 tritone and a 1200-cent octave). The change is a non-trivial 34 cents (a *sixth-tone* in traditional terms). Another way to consider the situation: The harmonic

barbershop seventh is substantially flatter than tempered; at the beginning of the song, the baritone is on the tonic; and the same note on “song” is now the seventh of a II7 chord. Either the baritone must flatten his tuning, or all three other parts will have to raise their tuning, or the chord will be far from the simple ratio. Audio File 14 demonstrates each of these options. Listeners should notice some distinction, though the differences are subtle. Most noticeable are the beats in the non-adjusted version. The beats are prominent, however, because vibrato is minimal and the tuning is held with consistency. The complexities of live voices might mask the tuning details here. In the context of the song at full tempo, the general ebb and flow of the harmonic blend will give a dynamic and effective character to the performance regardless of tuning precision. Furthermore, a chord out of tune from the harmonic ideal may still be functional. An out-of-tune chord will have more roughness, will be harder to sense with clarity, and won’t ring as well; but it will retain melodic, rhythmic, and some harmonic function as the roughness and tonal focus create tension which later resolves.

In demonstrating pure ratios in the main recorded examples and as shown in the graph, I have chosen to have the baritone flatten. The baritone part is not supposed to be singing a prominent melody but is supposed to blend into the harmony and listeners should mostly notice only the ring of the blended chord. The option of other parts shifting sharp sounds fine to me and may be preferred by some, but it does not fit the overall tuning of the phrase as well and would cause the tonal center to sharpen if strict tuning is otherwise maintained.

In the imprecise real-world context of a live quartet, a number of other compromises are possible. Perhaps the baritone will flatten slightly to approach the seventh but not completely reach it, thus creating an imprecise chord that has slower, more subtle beats than the one with no

tuning adjustment. Perhaps the other parts will tune just slightly sharp, making up the difference. Perhaps the tuning will shift throughout the duration of the chord, or perhaps the baritone may fail to adjust and the other parts may be slightly out of tune, maybe even flat. In that last case, the chord will have even more roughness. It is reasonable to hypothesize that lower-level amateur quartets may lack the control or sensitivity to be anywhere near accurate tuning. It is still possible, however, that the harmonic ratio is a conceptual goal toward which all quartets aim.

The II7 chord as a unit has, in theory, a root one perfect fifth (2:3 ratio) above the root of the V7 chord to which it eventually leads (ignoring octaves). In precise Just Intonation, the bass will be an 8:9 ratio (=2:3*2:3, divided by 2 to be within an octave) above the tonic to which it eventually resolves in m. 3. As that bass root is, in theory, 4 cents sharp of tempered, the tenor must shift 7 cents from its pitch on “more” to the note of the same name on “song” to maintain precise ratios. The average quartet probably cannot achieve this degree of precision. To the average listener, 7 cents is around a *just noticeable difference* for mid-range pitches without any vibrato or other masking. With even minimal vibrato, 7 cents is practically irrelevant melodically. In the primary versions of the accompanying audio, the 7-cent adjustment is used because some decision had to be made when specifying the computer-adjusted recording. Live quartets, however, need not be concerned with such tiny melodic tuning details.

The chord on “let’s” may be called G minor, but it makes more sense to instead call it a B♭6 (missing the fifth) because it is essentially a passing embellishment as the lead moves from the fifth through the sixth to the seventh of the ongoing B♭ chord. This triad would not make sense as a septimal minor chord (discussed later regarding m. 7). Here, the interval between the

bass and tenor is strong and it is clear that the baritone will move to an octave with the bass while the lead sings a perfect fifth below the tenor.

The chord on "sing" is simply an inversion of the first beat of the measure. In this case, the lead is on the barbershop seventh, and it may seem more awkward for the melody to be substantially flat of what appears as a tonic note in the score. However, if the baritone tuned flat at the beginning of the measure, then the sound of the blended chord will predominate. The lead will probably tend to match whatever tuning the baritone established. On the other hand, the same variety of compromises are possible as discussed for the first beat. As long as the chord blends well internally, the absolute tuning is not of major consequence. The lead may conceive of this note in two distinct ways: as the tonic for the overall key or as a barbershop seventh in the harmonic context. This is an example of a melody where tuning might be different depending on the presence (or lack thereof) of the harmony parts. Nevertheless, this pitch goes by quickly on its way to the next downbeat, so the tuning is not especially important at full tempo.

With a little explanation, the graph and ratio numbers offer valuable insights into the arrangement. Consider the unambiguous chord on "song," as shown in the graph in Figure 2. Without the baritone, the ratio of the other three voices is simply 2:3:5. The only element in the harmonic series between 3 and 5 is, of course, 4; and if the baritone were to sing that, his note would be a whole step higher. By singing the 7, the baritone puts the whole quartet in a one-octave-higher context of the harmonic series, making the 2:3:5 function as 4:6:10. This puts the fundamental of the entire series an octave lower, as indicated in the full chart in Appendix C.[23]

[23] Also see Appendix D for a general categorical chart relating the harmonic series to traditional scale terms.

On “let’s,” the lead moves to a note between his previous 6 and the baritone’s previous 7. There is no harmonic between 6 and 7, so if the baritone stayed on 7, the only fit for the lead would be an in-between in the next octave of the series, which would be harmonic 13. The resulting 8:13:14:20 harmony is interesting but definitely outside of barbershop style. By raising the baritone, we allow a tripling of the stable 2:5 bass-tenor harmony instead of only doublings. This means that the fundamental of the whole series is a perfect fifth lower than the chord on “song.” With the 2:5 functioning as 6:15, there is a more typical note for the lead at harmonic 10 and the baritone fits in at 12. Notice that here the ratio from bass to lead is the same as the previous lead to tenor.

If the harmonic series is a primary source of chord structures, then we can expect to see such adjustments in the parts when they do not fit well in combined low-integer ratio. Only by moving the baritone on “let’s” can the lead’s new note have a good harmonic location in the context of the overall harmony in the measure. Whether or not the 6:10:12:15 ratio is precisely tuned, I believe that the context suggests an approximation of that ratio over any other. Notice also that the chord on “sing” has the same ratios from “song” with only one pitch moved up an octave (doubled from 4 to 8). If a different voicing moved the baritone note on 7 down an octave to the bass, the rest of the ratio numbers would be doubled, so that would be a rougher choice. The suggestion here is that the structures of these chords are not based simply on the presence of pitch classes; the chords are based on a specific unified structure related to the harmonic series. The consistency with which barbershop follows this theory (judging by the present analysis among many others I have analyzed in the past) provides support for this idea.

Measure 2: passing half-diminished chord into V7

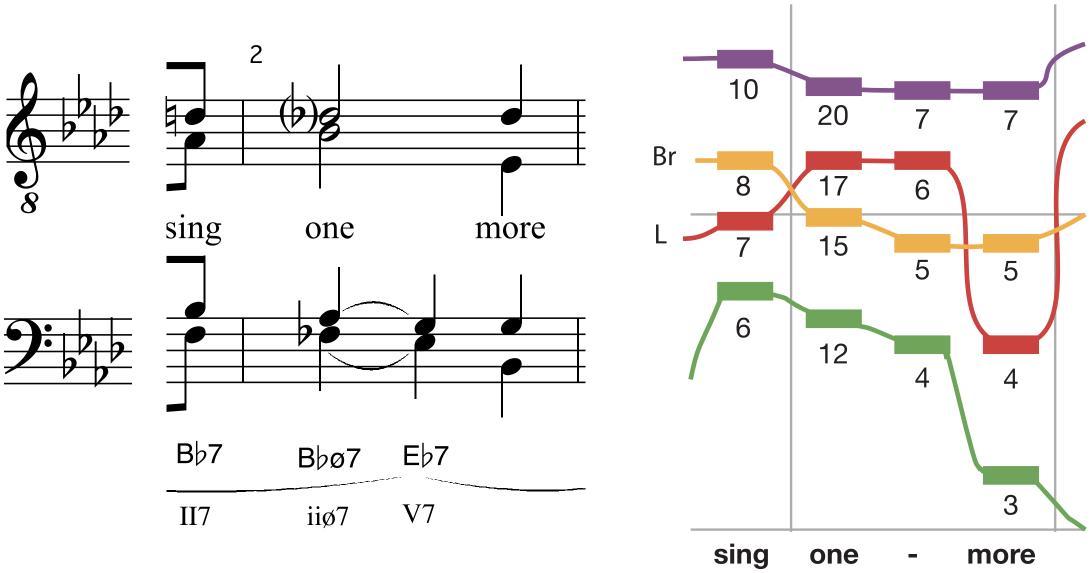

Figure 3. In measure 2, a passing swipe creates a half-diminished chord leading to the dominant.

Measure 2 is mainly the dominant V7 chord. The first beat has the bass and baritone on accented passing tones that could have been left out with no impact on any other chord. It is commonly stated that barbershop doesn't really have non-harmonic tones, and Liles was surely aware that the structure of the combined pitches on beat one create a half-diminished seventh chord (m7♭5 in jazz terms), one of the accepted chord structures common in barbershop. This chord can also be called a *minor six* (i.e. minor triad with an added major sixth) if a different note is selected as the root. In barbershop, this structure is found most commonly on the supertonic, as it is in this case (if calling it a m6 chord, the named root is then at the subdominant).[24]

The lowest possible ratio for the downbeat of m. 2 is 7:9:10:12, which is essentially a dominant ninth chord omitting the root. That tuning is usually the best choice for this type of

[24] See Appendix B for more discussion of the conventional chord naming and further alternative names for this chord. See Appendix D for a chart relating terms such as subdominant to other scale degree names.

chord, such as when the same notes appear on the downbeat of m. 14 (with one note simply displaced by an octave). In that later context, the chord resolves to the tonic directly. Here in m. 2, the next chord is V7 which only makes sense as a 4:5:6:7 ratio. The relation of these two chords presents some melodic challenges that may affect the harmonic tuning. Figures 3-6 show some different conceptions of this situation, and these are demonstrated in Audio File 15.[25]

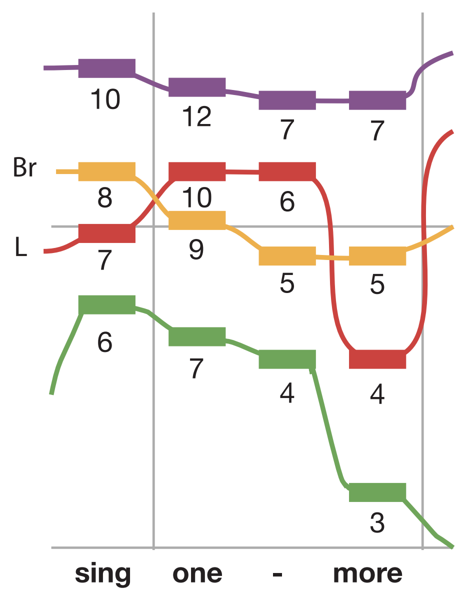

Figure 4. 7:9:10:12 tuning in m. 2 with the lead voice not shifting.

If the 7:9:10:12 ratio is sung on beat one and tuned to a lead voice that is stable into the next chord, as shown in Figure 4, this makes the bass voice quite flat, although that leads well in terms of contour. With this tuning, the minor third in the lead and tenor will change from a 5:6 (simplified from 10:12) ratio (316 cents) to the much smaller 6:7 (267 cents) on beat two (see

[25] One issue with presenting all these options in audio is that hearing lots of variations in tuning will lead listeners to generally have more flexible categorical conceptions for the chord. Ironically, the result of demonstrating the distinctions may be that the distinctions lose subjective significance. If a listener is primed with repeated hearing of only one option, then alternate options will be more noticeable and possibly even seem jarring.

also discussion of mm. 9-10 for the same issue in a different context). With the lead voice stable, the tenor will start especially high on the first beat and make an entire quarter-tone shift down to the barbershop seventh on beat two. Also, the baritone will be a comma sharp of the tonal center, and thus be singing the third distinct tuning so far in the song for the same notated scale degree, although it still fits fine in terms of the melodic contour.

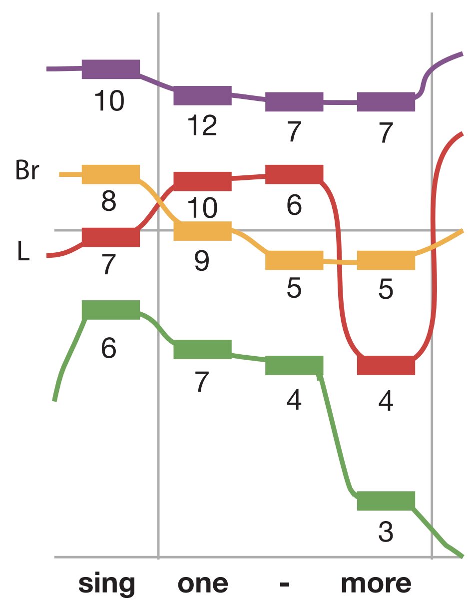

Figure 5. 7:9:10:12 tuning in m. 2, baritone on tonic, lead shifts

Figure 5 shows the same ratio as Figure 4, but tuned to have the baritone on the tonic instead of tuning to a steady lead voice. This option matches the preferred tuning for the chord of the same notation later in m. 14. This provides a better overall melodic compromise than tuning to the sharper lead. By tuning the baritone to the tonic, the lead sharps one comma (22 cents to be strict) from beat one to two, while the tenor flats one comma (27 cents). This splits the difference of the 5:6 to 6:7 interval change between the two parts with more subtle shifts than the

quarter-tone move in a single part. The bass is very flat here. To my ears, the result is novel but effective. It seems possible to get comfortable with this as a listener and might be learnable for singers, but it is not likely to be intuitive in this context because of melodic issues.[26]

Of course, as emphasized earlier, it is possible to have more complex adaptive tunings that change throughout the chord or perhaps have a mid-point compromise between the two absolute tunings.

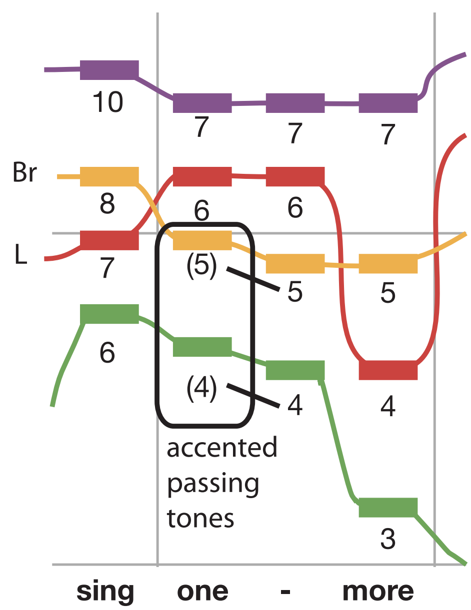

Figure 6. Measure 2, ignoring combined harmonic ratio on beat one

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[26] A barbershop theorist friend of mine feels that the tenor shifting flat in this context is awkward and should be avoided, regardless of whether it is the quarter-tone or the smaller comma shift. He feels strongly enough that he prefers for the tenor to simply sustain his tuning regardless of how it blends with the other voices. See additional discussion regarding mm. 9-10.

While I disagree both about the shift being unacceptable and about the degree of unlikelihood that singers may make such shifts, the idea brings up a potentially significant issue. Notice that if the 7:9:10:12 tuning were used as in Figures 4 or 5, and the tenor tuning is sustained into the next chord, the result means that the bass and baritone passing tones actually risk damaging the tuning of the following V7 (unless all three other parts were to shift sharper to compensate, thus sharpening the entire song's tonal center). Of course, the other tuning options discussed below (instead of 7:9:10:12) show little or no tenor shift and so allow the passing tones and a well-tuned V7 while still satisfying my theorist friend. But if a quartet were to carefully blend (with or without explicit intention) and come near 7:9:10:12 and then refuse to shift sustained pitches into the V7, then the impact of the passing tones would be significant. Because of this, some theorists might argue for alteration of the arrangement to avoid this potential problem. I agree at least that these situations can make the difference between easier, accessible arrangements or more complex, challenging arrangements. Thus, there are many reasons to consider omitting this swipe.

As illustrated in Figure 6, it could make sense for the baritone and bass to maintain the same 4:5 ratio that they will have in the dominant chord to which they resolve. The tenor and lead could stay on the same 6:7 ratio throughout their half-note, consistent with the written score. If the baritone is tuned precisely to the tonic, the combined chord ratio becomes 64:80:90:105. If the A♭ in m. 1 is tuned to the barbershop seventh and the baritone matches that on the downbeat of m. 2, the combined ratio at the passing-tone chord will be 84:105:120:140. If we allow for the swiped ratios to be flexible but strictly maintain the lead-tenor 6:7, then the smallest ratio possible is 21:26:30:35. That uses the thirteenth harmonic (26/2) which is considered outside of barbershop style. These are all complex, rough, and are clearly not as stable and unified as the 7:9:10:12 tuning. Of course, the non-harmonic function here is melodically effective, and so the ratio is unimportant in some regards. Many quartets probably conceive of this type of situation in this non-harmonic manner, without focus on tuning and blending for that moment.

The second lowest combined ratio (after 7:9:10:12) that fits the note structure at all is 12:15:17:20.[27] Because I used this tuning in my full recording, it is the choice illustrated in the full pitch graph in Appendix C and in Figure 3. This tuning does not have an especially low-number ratio and definitely is not as strong and blended as 7:9:10:12. Still, 12:15:17:20 is debatably blended enough to create a somewhat cohesive harmony, and it fits the melodic contour here. The tuning preserves the 4:5 ratio of the baritone and bass across beat one and two. The lead-tenor ratio changes from 17:20 to 6:7 (a very small 12-cent difference). The melodic shift in the tenor fits the overall contour. This contrasts with the larger 49-cent and 27-cent shifts implied respectively in the two 7:9:10:12 options shown in Figures 4 and 5.

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[27] This chord could be called a major-seven chord with an added flat-ninth, missing the root (i.e. M7♭9-no-root), with the missing root in this case on A.

Using 12:15:17:20 and the lead consistent across the two beats, the result is a nearly-tempered bass pitch, the baritone just 13 cents flat of tonic, and the tenor slightly flat of tempered before flattening just slightly further in beat two to be on the barbershop-seventh of the V7 chord. The tuning is actually quite close to the concept in Figure 6 that ignores the harmonic ratio in favor of a non-harmonic consideration. The only notable difference is the slightly sharper tenor, which works well because the note is the first D♭, the fourth scale degree, and the tuning will be comfortable to sing closer to the normal diatonic pitch before relaxing just slightly into the barbershop seventh. The level of tension is appropriate here and resolves beautifully into the dominant chord. Quartets may conceive of the situation as non-harmonic tones, but the result could arguably approximate 12:15:17:20 enough to imply that ratio.

I hear the 12:15:17:20 option as sounding most like the harmony I expect a live quartet to have in this context. On the other hand, 12:15:17:20, even tuned perfectly, might be heard by some people as a complex approximation of 7:9:10:12; or perhaps all of the options are heard as somewhat complex and rough with some listeners lacking any sense of clearly unified harmonic tuning. While no live singers are precise to any of these ratios, one or another ratio may be the one most implicitly conceived and approximated.

Continuing on in the song, the V7 on beat two is clearly best as a simple 4:5:6:7 ratio, then inverted on beat three to 3:4:5:7. There is nothing ambiguous here and no other ratio makes any sense. The closer a quartet is to this tuning, the more unified and blended the sound will be. Of course, even poorly tuned seventh chords will still have effective tension that will resolve at the tonic in the next measure. However, this is a case where the style's focus on seventh chords, lock, ring, and blend makes it desirable to tune the chord accurately.

Aside: Overview of comma drift and compensation for circle progressions

To understand the issues in measures 3-4, an overview of *comma drift* is necessary.[28] A *comma* is a small interval that is the difference in tuning between two pitch intervals of very similar size. Commas are prominent in tuning theory where notable comparisons are made between various similarly sized ratios, such as 8:9 versus 9:10 (a 22-cent difference).

As stated before, subtle tuning precision is more perceptible harmonically than melodically. Shared pitches across chords can be mildly adjusted with no ill effect, often completely unnoticed by listeners. This masking of tuning adjustment occurs partly because the human voice is imprecise with pitch always gliding subtly in complex ways along with changing timbrel qualities. A very clean, precise sound with strong attack and consistent tuning will draw more attention to small tuning shifts (though that may remain acceptable to familiar listeners).

Strict Just Intonation theory suggests that tuning from chord to chord is influenced by sustained pitches. For example, the perfect fifth of a tonic chord would be assumed to match the root of the dominant chord. I submit, however, that small melodic shifts may be used at times, as discussed in this analysis, and are acceptable and barely noticed; so it will, therefore, also be acceptable if the tuning of the root of a dominant chord is a few cents off from the fifth of the tonic chord — even if there is no particular reason for such a shift, as long as each chord is tuned well internally. The limitations of fixed pitch instruments are almost certainly the impetus behind the development of rigid tuning theories. With voices, or with fretless, keyless instruments, there are many acceptable and varying possibilities. Nevertheless, strict Just Intonation has been applied to barbershop theory in the past and I will address the issue here.

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[28] Wild & Schubert (2008), available online at musicstudies.org, has an illustrated discussion of this issue in a similar vocal context complete with audio examples.

With precise integer ratios assigned to every pitch interval, the absolute tonal center of a piece will drift by a comma in certain progressions. For example, a chain of four perfect fifths (2:3 to the 4th power = 16:81) creates an interval one comma larger than the interval from fundamental to fifth harmonic (1:5 = 16:80), a difference of 80:81 (about 22 cents).

If the major third of a tonic chord is tuned to the fifth harmonic and then that same pitch is used as the root of a III7 chord in a strictly-tuned circle progression, the tonic at the end will be one comma flat of the original. To compensate for this, Jim Richards (2001) suggests that quartets should learn to anticipate the progression and, upon leaving the tonic, make a one-time adjustment, sharpening the root of the new chord by a comma. For either VI7 or III7, this adjustment requires a shift of the note that had been the major third of the tonic and which is often sustained in a single voice. Nevertheless, the shift is musically effective because it adds energy and excitement which is appropriate when departing from the tonic. The tension the shift causes is then released as the progression eventually resolves to the tonal center.

How quartets deal with this situation is a matter for further research. Live singers are not precise to the cent and will almost certainly vary slightly from chord to chord. Richards' suggested adjustment may be approximated, however, by singers learning to sharpen slightly-but-noticeably whenever they leave a tonic to start on a circle progression. Alternatively, it is possible that quartets adapt throughout the progression with continuous subtle sharpening. Perhaps some quartets do drift flat or are otherwise imprecise through the progression but then adjust as necessary at the end to arrive on the original remembered tonic. It is also possible for some quartets that such progressions do cause the tonal center to drift flat.

My personal experience studying these issues over the years is worth mentioning. In my studies, I became accustomed to hearing the precisely tuned recordings that I created (like the examples included with this treatise). Initially, I followed the advice of Dr. Richards and adjusted sharp to compensate for circle progressions. However, when moving to a relative minor (the vi chord), especially when not part of a circle progression, I opted instead to maintain the root and third from the tonic chord and not make any comma shift.

On one hand, the lowest possible ratio for a minor triad is 6:7:9, which is identical to the top three notes of a ninth chord. That tuning works smoothly when moving in a circle progression, such as from ii to V7. It is as though the first chord is a part of a V9, but without root and third, and then we simply move to V7. The 6:7:9 tuning works similarly for vi to II7. On the other hand, the vi shares two notes with the tonic chord, so shifting from tonic to a 6:7:9-tuned vi chord may be awkward. When the vi leads directly to V or to IV, it then makes sense to sustain the root and major third tuning from the tonic chord, resulting in a vi chord tuned to 10:12:15. Though the 10:12:15 minor tuning has higher combined numerals, the dyad intervals (the ratios among note pairs) contain slightly lower numbers than those of the 6:7:9 minor.

In my studies, I have considered the two minor triads to be distinct chords with distinct qualities. Internally, they have entire quarter-tone difference in the tuning of the middle note of the ratio (the bottom 5:6 dyad of 10:12:15 is a minor third interval 16 cents sharp of tempered while the 6:7 of 6:7:9 is a minor third 33 cents flat of tempered). While the 6:7:9 chord may be considered a dominant ninth without the root or third (as mentioned above), the 10:12:15 chord is a major seventh chord without the root. To me, the 6:7:9 variety feels like a dominant, bluesy, jazzy sound; while the 10:12:15 has a more melancholy, classical, relative-minor sound.

In recordings that I made, I had to consider when tuning a III7 chord whether it was leading to a 10:12:15 relative-minor vi or whether it was part of a circle progression (which would most commonly lead to VI7). I originally decided that a I-III7-vi should not shift, and so the third scale degree would be maintained consistently through the progression. In contrast, for a I-III7-VI7 progression, I would shift the III7 one comma sharp, following Richards' suggested compensation for comma drift. The internal tuning of the III7 remained the same in both cases.

I discovered that I could clearly hear the distinction between the two III7 options. I tried blind tests on myself (with help from friends) with playback stopping at a III7 sometimes tuned to the third of the tonic and sometimes one comma sharper. I found that I consistently identified the difference (when played in context) and felt the flatter III7 to be more connected to tonic, and more minor in quality, with a definite expectation of hearing the relative minor next. I even experienced the emotional and expressive associations of a relative minor. Conversely, when I heard the sharper III7, I expected the whole circle progression with barbershop seventh chords, and I inferred a jazzier mood in the music.

I wondered whether my perceptions were of something universally inherent to the chords. The flatter III7 does contain more consistent melodic pitches, while the sharper comma-shifted tuning adds energy and a different feeling to the music due to the shift itself. Today I suspect that my experience was largely due to learned syntactic associations. Obviously, further empirical testing with various listeners is warranted; but my personal experience was consistent, unambiguous, and undeniable. This might suggest that it is possible for music to have tonal categories at least as small as a comma. Another hypothesis is that it is possible to develop a musical syntax schema based on audible comma shifts.

Questions arise with a vi-II7-V7-I progression regarding tuning and comma drift. The vi chord could be tuned as the 10:12:15 relative minor; but that has a root 16 cents flatter than tempered, so sustaining common tones into the II7 could cause comma drift. One possibility would be to treat the vi as a tonic substitute and then use Richards' suggested compensation when moving to the II7. Otherwise, drift could be avoided with the vi tuned to 6:7:9.

Significantly, tempered tuning creates a compromise between these options and so allows approximation of either one, perhaps with a different implication upon arrival than on departure from the chord in question. The ambiguity of temperament provides this unique function but the distinctions of the Just tunings are lost. Live singers may not sing strict mathematical equal temperament, but they certainly can (though do not necessarily) make compromises that effectively temper these comma differences.

It should be remembered that, in addition to these and other alternative adjustments, even drifting flat may be acceptable, depending on the listener and the other details of the context. In testing the options with my tuned recordings, I personally find all these possibilities interesting and acceptable. I am inclined to prefer the options I became familiar with during my early studies, but I now acknowledge how flexible and variable the music can be, particularly with the imprecision and inconsistency of live singers. As we proceed through the present analysis, keep in mind that this complexity arises from ambiguous situations. Some situations, by contrast, are unambiguous and offer far less varying interpretations and tuning possibilities.

Measures 3-4: Initial tonic resolution, III7, and the fully diminished chord

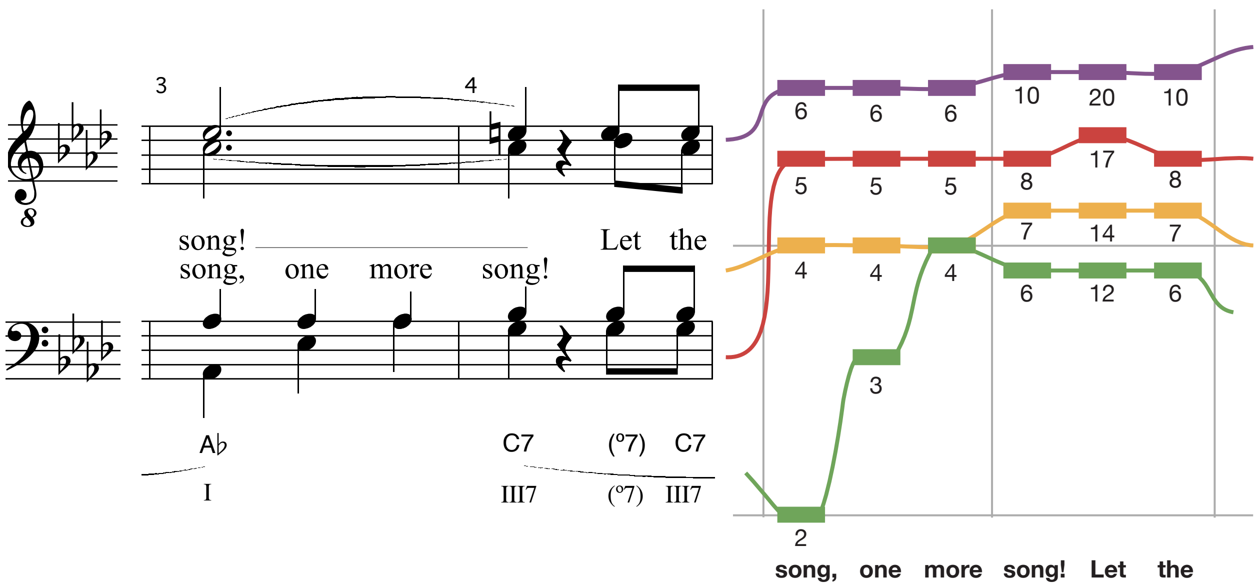

Figure 7. At the end of the first phrase, the tonic in measure 3 moves on to III7 in measure 4.

Measure 3 in *One More Song* contains the first strong, stable, root-position tonic chord. This chord is the focal point of the II-V-I circle progression of the initial phrase. The bass then simply moves up through the harmonic series.

Then we come to the issue of the III7 chord. Because the chord in m. 5 is the minor vi, it seems best to keep the lead note consistent from m. 3 into m. 4 and on into m. 5.[29] One result of this choice is an especially flat baritone note. Tuned strictly, the lead's major third in the I chord is already 14 cents flat of tempered, and the baritone is on the barbershop seventh of that; thus, the baritone B♭ here will be a quarter-tone flatter than the B♭ in m. 2. This is not a problem for a baritone comfortable with adjusting to harmonize to the other parts, but it does highlight the type

[29] Audio File 16 demonstrates the difference between this I-III7 with a steady lead and with a comma sharper shift. Though the shift isn't necessary in this context, this is an opportunity to demonstrate (though without the ideal contextual syntax) the distinction discussed earlier regarding my personal experience with tuning the III7 chord.

of challenges that beginning barbershop singers may have if coming from a background with tempered instruments and terminology.

The word “let” in m. 4 has a simple neighboring movement in the lead. This is another example where determining a precise tuning is less important than acknowledging the general tension and release of a non-harmonic melodic contour. If we insist on naming every possible chord, this is a *fully-diminished seven*. Because tempered tuning eliminates subtle differences between intervals, fully-diminished chords are traditionally described as stacks of identical minor thirds and any note may be named as the root. The best lowest possible ratio for a diminished chord in close position is 10:12:14:17, which can be a somewhat blended sound. Unlike tempered tuning, the Just Intonation diminished chord has minor third intervals of all different sizes: 5:6, 6:7, 14:17, and (adding the octave) 17:20. The context of the harmonic series identifies this tuning as a barbershop seventh chord with an added flat-ninth (conventionally named simply a *flat-nine* chord) missing the root. The ratio reveals how a diminished chord is simply a seventh chord with the root moved up a semitone (or with the rest of the parts down a semitone), as represented in the by the difference from 16 to 17. This analysis of the diminished chord as an altered seventh chord fits the most common contexts both in barbershop and in jazz and popular music. This insight shows why the best precise tuning on “let” is 12:14:17:20. The lead simply moves from 16 to 17 and back (notice that doubling 6:7:8:10 gives 12:14:16:20). Still, because the note here goes by quickly and the lead is the only part moving, I am certain listeners and singers will mainly hear the melodic shift as a non-harmonic tone and pay little attention to the tuning of the chord.

Measure 5: Relative minor, and m7 versus M6 chord names

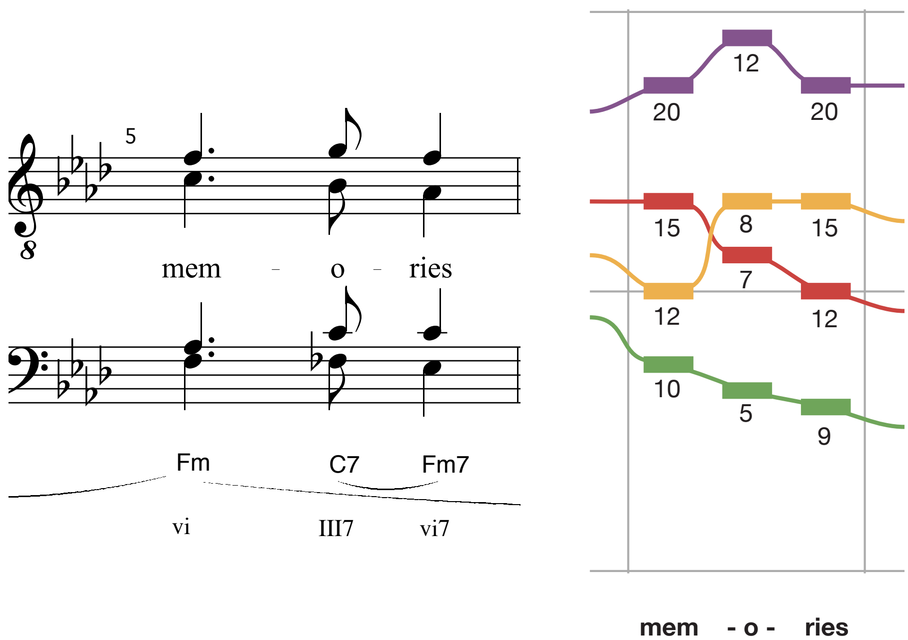

Figure 8. The relative minor and a chromatic bass line in measure 5.

As discussed earlier, the relative minor chord in m. 5 (see fig. 8) is usually best tuned to 10:12:15. The baritone and lead stay in the same 4:5 ratio they had on the tonic in m. 4, and the bass must tune a 2:3 perfect fifth below the lead. The embellishment in the middle of the measure is simply a return to the previous III7 chord from m. 4 with a simple octave swap between bass and tenor (and exact swap between lead and baritone).[30] The final chord adds a seventh to the main chord, making it a *minor seven* chord. Notice that this chord involves the same three upper pitches as the downbeat (though baritone and lead are now switched), while the bass moves to a pitch that fits exactly in the same harmonic series on the next lower number.

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[30] In an older printing of *One More Song*, the common identity of the III7 chords in measures 4 and 5 is more explicit, having identical note names. The more recent printing, as here, uses the enharmonic F♭ to make the melodic leading more clear and avoid the need for excessive accidentals.

It is worth mentioning that there is no tuning difference between this minor seven chord and a major-six chord. In this case, one could call the example either Fm7 or A♭6. Those names are of no important consequence. One name implies a focus on the minor aspect of the chord, the other name focuses on the major root. There are cases where one name may make more sense in context, but the tuning may still be identical. It is also worth noting that a such chords contain two pairs of perfect fifths (disregarding particular voicing), with a minor third between the roots of the pairs. Perfect fifths are clearly 2:3 ratios without any reasonable alternative. The minor third between the pairs of fifths, however, may be 5:6 (as in this case) or possibly 6:7 (see later discussions of m. 7 & m. 11).

Measures 6-7: Parallel movements and tuning the ii chord

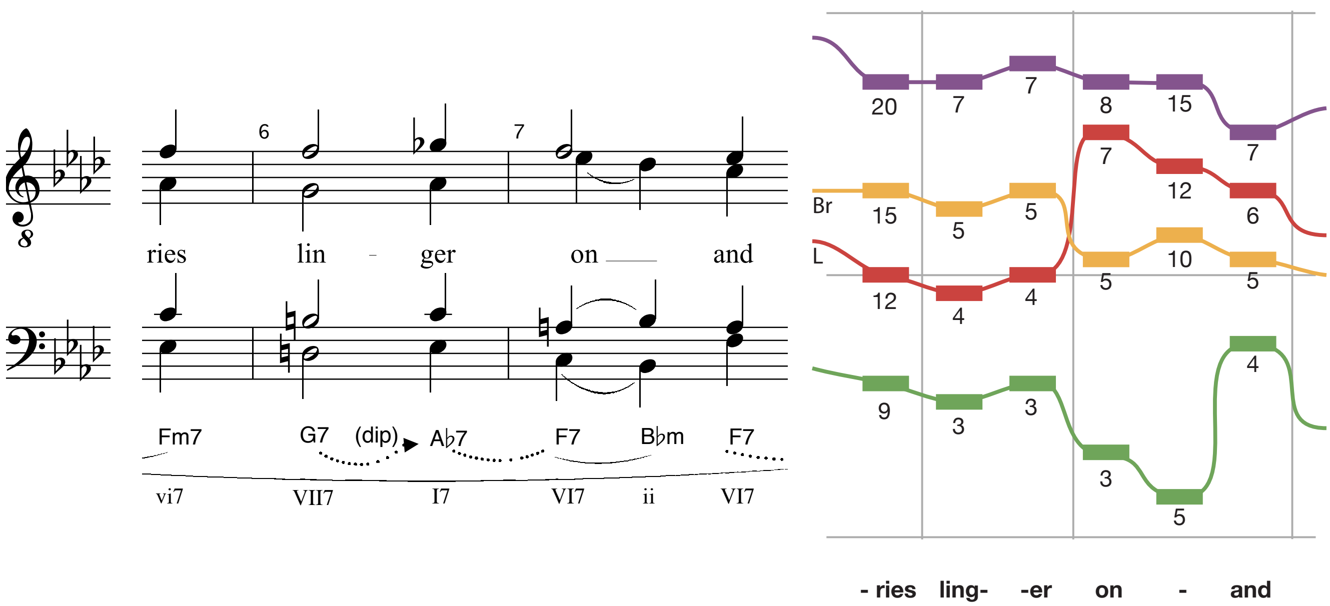

Figure 9. Continued extensions and embellishments of the VI chord through mm. 6 and 7.

In m. 6, we can see one value of using the alternative A♭6 name for beat three of m. 5. Realizing that there is first a major triad with an added major sixth, one can see that the triad moves down as a unit until the major sixth becomes a barbershop seventh. Thus the 9:12:15:20 chord in m. 5 becomes a 9:12:15:21 chord in m. 6, which simplifies to 3:4:5:7.[31] The resulting chord is rooted one half-step below the tonic (a small 87 cent half-step in precise Just Intonation). The entire chord — including the seventh — then shifts up to be rooted on the tonic. The graph in Figure 9 makes especially apparent the effectiveness of this wonderful progression. In many similar dips (as I call them), melodic tuning from chord to chord is of no precise importance (see m. 10); but here the tenor connects the first two chords, and the next chord makes sense to be tuned to the tonal center. Also, notice that the chromatic shift in this rhythmic context is a particularly effective device for the word “linger.”

Measure 7 starts with a barbershop seventh chord with the internal tuning 3:5:7:8, which is known in the barbershop world as a *Chinese seventh*. With its 7:8 large whole tone, it has a unique sound that is distinct from other barbershop seventh structures that keep the seventh and root further apart. The blend of this chord and its prominent use are further evidence that barbershop is based on combined ratios rather than on issues of independent intervals. There are only certain cases where such small harmonic intervals are common and they consistently fit the context of low-number ratios.

Those unfamiliar with the song might expect this VI7 to be the beginning of a circle progression, and it does move next to a ii chord. Looking ahead, however, one sees that this progression resolves without coming to the V chord. Given this context, it makes some sense for

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[31] On a side note, this same 20:21 shift is the best underlying harmonic tuning for any context in which the major sixth and minor seventh are heard back and forth against a chord, as occurs regularly in blues and jazz, only in those cases the upper note moves while the triad is stable.

the root of the VI7 to match the root of the vi chord from m. 5 whose root has been consistently reinforced in the tenor. That is the tuning is used in the example recordings. The final chord in m. 7 is simply a re-voicing of this first beat.

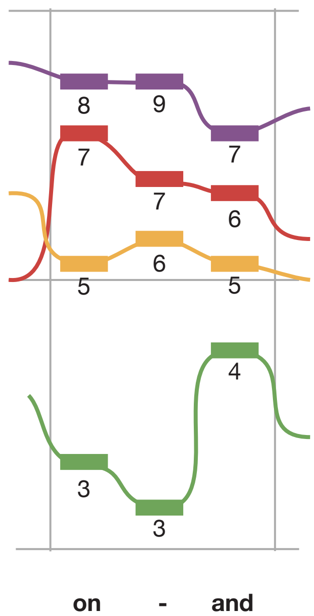

Figure 10. Alternate tuning for m. 7

The second beat in m. 7 is a simple minor triad. The tuning could be 3:6:7:9 (see Figure 10), which is not only the lowest possible ratio but also maintains the relationship of the bass and lead throughout the swipe from beat one to beat two. This tuning works excellently as the lead-bass swipe simply re-harmonizes the sustained tenor. If the baritone also swiped down with the other parts, the result would be a 3:5:7:9 (a dominant ninth chord without the root). Instead, the baritone swipes up to the 6 ratio, resulting in a simple triad (in a tuning which, once again, implies a dominant ninth harmonically but lacks both the root and the major third). One issue is that the combination of lead on the 7-ratio and the overall tuning of this measure (discussed

above) results in the lead voice being an entire quarter-tone flat of tempered — and of the pitch he sings on “let” in m. 4. If the lead instead sings a more diatonic, common pitch, then the chord ratio will be 5:10:12:15 (as in fig. 9), which is not as blended but is acceptable and matches the tuning of the vi chord in m. 5.

As discussed previously regarding minor triads, because the octave and fifth relationships of the other parts are fixed without question, the particular ratio is determined by the minor third note, in this case in the lead voice. If the whole measure were sharpened, it would increase the possibility that the lead would tune to the flatter 7-ratio on the second beat. This situation is definitely a case where real quartets might temper their tuning, maybe only roughly approximating the theoretical tunings. I have selected the 5:10:12:15 tuning for the main recording and graph. Audio File 17 demonstrates both tunings along with a half-way compromise (which I tuned to 8:16:19:24 utilizing the nineteenth harmonic which is a near-tempered minor third four octaves from the fundamental). To me, the distinctions are perceptible though subtle, and all three options are acceptable but vary in expressive quality. Minor chords clearly offer far more harmonic tuning possibilities than major chords.

It is worth noting some further contextual distinctions of these tunings. The 6:7:9-based tuning is strongest in root position with the root doubled, as here in m. 7, while any other inversion of that tuning will be rougher. On the other hand, the 10:12:15 tuning functions well with the root in the bass but may be as blended or perhaps better in first inversion, depending on exact structure and octave placement. Also, only the 10:12:15 tuning makes sense to be named a major-six chord. Finally, the tuning using the nineteenth harmonic is best in root position or second inversion and ideally has the mediant (the 19) in the highest possible octave.

On another interesting point, the likelihood is low that the lead and tenor singing a duet would tune their major third to 7:9 (435 cents) when they could sing the simpler 4:5 (386 cents). Adding the root of the minor triad makes the 7:9 ratio more likely (especially when the root is reinforced at the octave as in m. 7). Thus, the presence of the entire quartet may alter the tuning of two voices compared to the two as a duet.

Measure 8: A more common half-diminished chord and an augmented chord

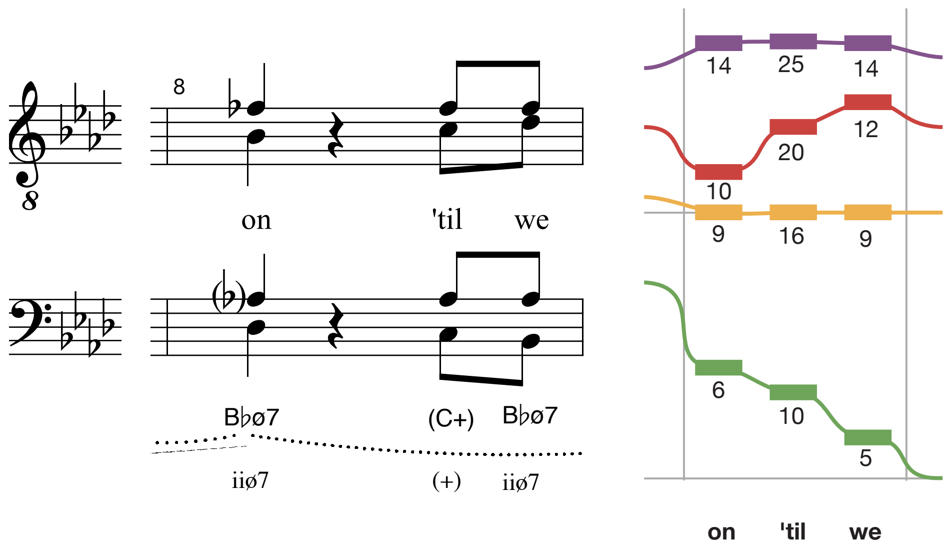

Figure 11. The embellished half-diminished chord in measure 8.

Measure 8 contains the same half-diminished chord as in m. 2, except with the bass and tenor notes swapped. Therefore, the same two options exist for potential tuning. There is not, however, a resolution to V7, nor any swipes. This situation is not a case of non-harmonic tones. Though all the tunings discussed for m. 2 are possible, there is no reason not to simply use the lowest ratio 6:9:10:14, making this essentially a ♭VII9 with a missing root.

The chord on beat two is an augmented triad. The lowest possible ratio that could arguably by called augmented is 7:9:11; but any of use of harmonic 11 is considered outside of barbershop style, and the tuning is unlike the traditional augmented definition because 9:11 is a *neutral third* half-way between major and minor (though the 7:11 interval is only slightly flat of a tempered augmented fifth). In this context, the best combined ratio based on that tuning would be 7:11:14:18, which would require extreme sharping of the tenor and slight flattering of the baritone if the lead and bass stay on their already somewhat flat pitches. The next lowest triad ratio that is arguably augmented is 8:10:13, which here would be 5:8:10:13. Though that tuning is the lowest combined ratio possible in this context (and sounds excellent, in my opinion), harmonic 13 is also considered outside of barbershop style. The thirteenth harmonic is 41 cents sharp of tempered, could be considered a *neutral sixth*, and the 10:13 ratio is over a quarter tone larger than a tempered major third (and therefore even larger still compared with a 4:5 major third). With 5:8:10:13 here, the tenor is again extremely sharp, but the other voices fit perfectly well. It is very unlikely that any quartet would sing either of these odd tunings, although I have included them in Audio File 18 for interest.

Instead of the augmented chord, the tenor could have simply been a half step lower, resulting in a first-inversion tonic chord. Or if the tenor moved up to F, it would be a return to the earlier vi chord. Those options would be more typical because augmented chords are usually avoided in barbershop arranging. Either of those solutions might be chosen over the augmented chord if a quartet was to work out the situation by ear without a written arrangement.

Without altering the notes in the score, the best tuning involves moving another octave up in the harmonic series from the 5:8:10:13 to 10:16:20:25 (tenor at 25 instead of 26=13*2), the

tenor is at a simple 4:5 ratio from the lead. The result matches the conventional definition of an augmented chord as two identical stacked major thirds. With this tuning, the tenor is a negligible 7 cents sharper here than on the surrounding pitches (the tenor moves from 9:14 from tonic to 16:25, which are almost identical). The 10:16:20:25 ratio is the most comfortable tuning here, and the particular context fits the augmented chord remarkably well.[32] The result is still not particularly well-blended, and the chord goes by so fast at normal tempo that listeners will mainly perceive non-harmonic voice-leading into the strong tonic chord in the next measure.

Measures 9-10: Tonic resolution to a circle progression, "slimy" tenor shift

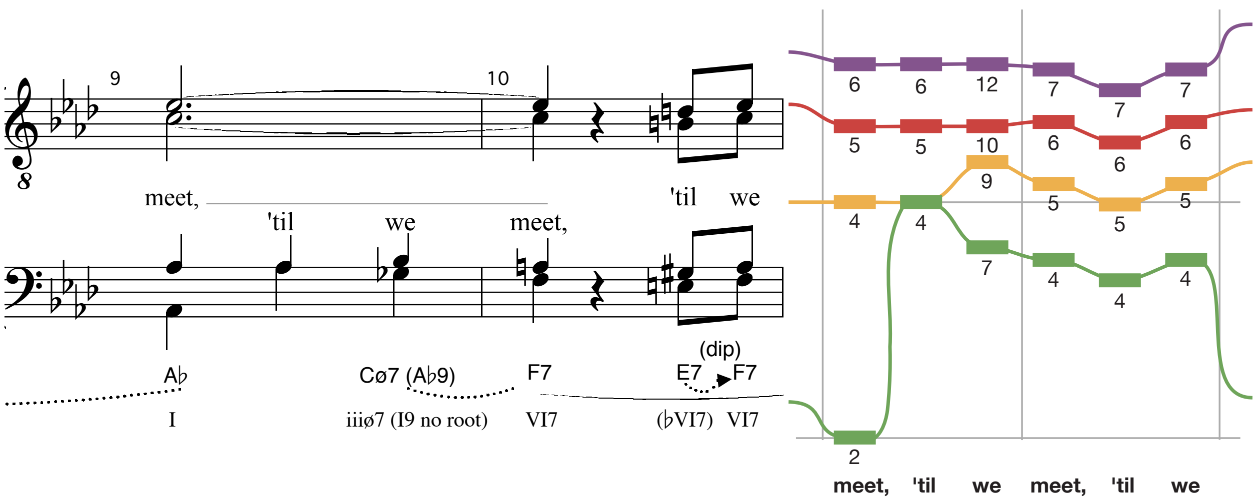

Figure 12. Tonic into VI7 starting a circle progression in mm. 9-10.

The simple tonic resolution in m. 9 quickly moves on to what could be called a half-diminished chord (see discussion of m. 2). In this case, it is especially clear how it is a ninth chord with the root missing, which then leads to a VI7 in m. 10. As discussed earlier regarding

[32] One final ratio possibility for the augmented triad is 12:15:19. While that ratio is the closest one to tempered tuning, using it in this context would give 12:19:24:30, a higher combined ratio than any other option mentioned, and would, like the 11 and 13 tuning options, require the tenor here to sharp by a comma at the augmented chord.

tuning options in m. 2, there again is a sustained minor third harmony changing from 5:6 to 6:7. There is also the issue of potential comma drift through the upcoming circle progression.

This situation is a common example that is typically discussed by barbershop theorists. I have heard it suggested that the the tenor should simply stay on the original pitch (see footnote 25), thus creating a 20:25:30:36 chord (compare to 4:5:6:7 = 20:25:30:*35*) in order to avoid a comma shift during the sustained note. The justification given for avoiding the shift is primarily a dislike of the sound. Some people describe prominent comma shifts as awkward and “slimy” sounding (based on personal discussions). However, I think the issue is one of pedagogy and expectation. Were this written as two different notes, perhaps it would be treated like the tenor in m. 3 to m. 4 where there is explicitly a change of pitch, and then it would be more accepted.

Distaste for the small shift may also be an issue of categorical conception. Perhaps a certain small amount of shifting that stays within a conceptual category for tonality is awkward, while, in contrast, the shift in m. 3 to m. 4 is large enough to be clearly a categorical change. The “sliminess” that some people describe of larger comma and quarter-tone shifts might be due to *cognitive dissonance* regarding an audible change that is not large enough to move to a new conceptual category. Analogously, in spoken language we have both single vowel categories and diphthongs that change from one category to the next; but an abrupt change entirely within a single vowel category might elicit this same slimy feeling. In other words, a change is definitely perceived, and yet the listener lacks any meaningful way to identify the shift in their categorical understanding and just feels baffled and uncomfortable as their implicit expectations of the musical system are violated. However, as both speaking and singing voices are highly variable and make smooth rather than abrupt shifts, I believe this issue does not arise in practice.

The history of tuning theory, especially related to fixed-pitch instruments, is full of debate about this subject. I had such experiences of slimy, uncomfortable shifts with microtonal recordings when I first studied this subject. However, my feelings of repulsion diminished as I became familiar with these quarter-tone and comma intervals. Eventually, they became part of my internal schema, and I now fail to hear anything weird in them. Furthermore, I would be shocked if any listener still hears comma shifts negatively (if perceived at all) in complex contexts with smooth gliding, vibrato, harmonic blend, and repeated reinforcement leading to familiarity and habituation. I think these shifts are unnoticed by most, and the main perception is of how well the harmony blends.

Due to the notation problem, however, it is likely that some quartets do not adjust tuning when comma shifts are otherwise called for. I believe that if a quartet learned to read my pitch graph instead of the Western notation, they would see the subtle shifts and sing them with comfortable intention.[33] Finally, keep in mind that poor tuning can be part of acceptable musical contrast if it later resolves to a blended chord, despite insistence in barbershop pedagogy that all chords should blend. Questions of the tolerance and significance of these issues calls for further empirical research.

The compromise I have chosen in the accompanying recordings and shown in the main graph is to make the overall VI7 chord in m. 10 sharpen one comma. This adjustment requires the lead voice to sharpen by one comma from the previous chord while the tenor flattens by one comma. Thus, instead of a quarter-tone shift in the tenor, the lead shares the responsibility for

[33] I am not necessarily suggesting that my graph is a great tool for the average singer, though it has pedagogical value. Of course, neither do I want to rule out adapting it for direct use in arranging if it could be helpful.

changing the 5:6 into a 6:7. This tuning puts the bass and baritone on comfortable pitches, and the shift compensates for comma drift in the circle progression.[34]

At the end of m. 10, there is a quick dip in all four parts at once, followed by a return to the same chord. This dip requires no specific melodic tuning. As long as all the parts move the same amount to retain the internal harmony and the dip is within the general category of a semitone, it will be effective.

Measures 11-12: Minor and minor-seven chords within a circle progression

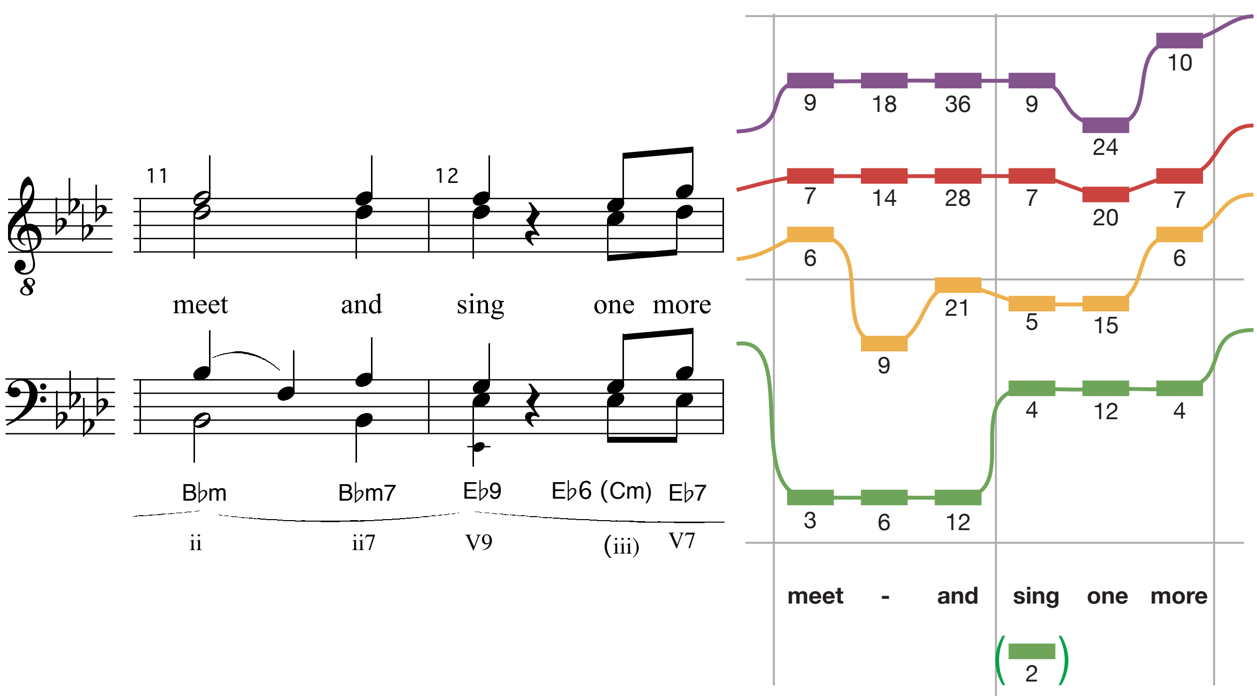

Figure 13. Baritone embellishments over the ii chord leading to V9 in mm. 11-12.

[34] Despite my use of pure harmonic ratio tuning, some research suggests that major thirds may be tuned substantially sharper (Eskelin, 2005; Hagerman & Sundberg, 1980; Nordmark & Ternstrom, 1996). I suspect that preference for sharper major thirds is almost entirely due to encultured familiarity with tempered tuning (Loosen, 1995; Hahn & Vitouch, 2002). Significantly, however, cultural tuning preferences are varied particularly in the case of sounds like the human voice which exhibit random flutter that masks acoustical beats (Hagerman & Sundberg, 1980; Nordmark & Ternstrom, 1996). In *One More Song*, if the lead were tuned sharp on the major third during the tonic chord in m. 9, then he would not need to shift to be on a good pitch for the fifth of following sharpened VI7 chord.

In m. 11, the initial chord is the same notated minor triad as in m. 7. Again, the tuning of the lead affects the choice between 3:6:7:9 or the less blended 5:10:12:15; but in this case, the lead is approaching from below and is not passing on to a next note. Thus, the lead should have no trouble tuning to the flatter 7 tuning. Furthermore, that option allows for consistent lead pitch across mm. 11-12. The 3:6:7:9 tuning also has harmonic implications consistent with the V chord to which it leads. When the baritone swipes down, he arrives at a perfect fifth with the bass and octave with the tenor which necessarily doubles the overall ratio numerals.

On beat three, the baritone moves to a new pitch a minor seventh from the bass and a perfect fourth from the lead. With the other parts not changing, the sensible tuning here is for the baritone to be a simple 3:4 ratio with the lead and also 4:7 with the bass. Though all the dyads are simple (arguably a very significant fact), the combined chord ratio is 12:21:28:36, which is a higher combined ratio than any other selected overall in this analysis. Looking into other options, the lowest possible ratio for this chord is 5:9:12:15, which is a re-voicing of the vi7 in m. 5. Out of context, that harmony would be fine, but here it would require the lead to shift up a quarter tone on "and" and back again on the next chord (assuming the 3:6:7:9 tuning at the beginning of the measure). The 5:9:12:15 tuning would also put the baritone up to a pitch a comma sharp of the tonic (assuming the root of the ii is a perfect fifth above the V). If m. 11 were tuned to start with the 5:10:12:15 minor tuning, then "and" could be tuned to 5:9:12:15 without requiring any shifts in that measure; but the middle chord would be a high 10:15:24:30 ratio and the lead would still shift downward when moving to the next measure (assuming a well-tuned V chord ratio). A stranger possibility, using the preferable 3:6:7:9 at the beginning of the measure, would simply have the baritone sharper on "and" to make a 6:11:14:18 ratio, thus fitting the same

harmonic series as the chord on beat two. However, 6:11 is a quarter-tone *neutral seventh*, a foreign sound to barbershop. Another novel option is the lead on a 19 ratio, giving 8:14:19:24 on "and." The same shifting in the lead would occur as with the 5:9:12:15 option, but the shifts would be only a comma instead of a quarter-tone. The 19 option could be used across the measure or just on the single chord; and it has the advantage over the 5:9:12:15 option of maintaining the 4:7 interval between baritone and bass. Finally, all manner of compromised tempering is possible, including approximations of all these chords and the possibility of the dominant in m. 12 being rougher with a 7th that is sharp of harmonic. The significance of all these different options is a matter worthy of further discussion, and I expect opinions are reasonably varied. All these options involve similar types of issues to those demonstrated earlier, so I have opted not to invest time in creating audio examples of all these alternatives.

Also, standing position of the singers could be a factor here. Typical quartet formation has the baritone on the outside of the quartet, next to the bass and farthest from the tenor. In the chord in question here, the baritone could tune to the bass next to him and not pay as much attention to the other singers. This brings up larger questions about whether tuning is more based on dyad pairs or on a combined whole. As mentioned earlier, all the dyads in the 12:21:28:36 option are low ratios, and that tuning provides the very best melodic context for all the parts and for the general progression. Another way to consider this tuning is as a 6:7:9 minor triad with an an added minor seventh a 2:3 perfect fifth from the 7 part of the ratio. As the 6:7:9 chord itself is the upper part of a dominant-nine, this version of a minor seventh chord is a further extension of that. In other words, these pitches are extended notes that have the feel of the V9 which follows.

Once again, one can see that some situations are more complex and vague than others. Perhaps m. 11 is an example where woodshedding might have altered the arrangement. The baritone could instead simply stay on the lower pitch from the second beat or return to the first beat pitch. Maybe an entirely different chord could be chosen for the third beat.

Regardless of all these tuning issues, this baritone *cherry* (as it is called in the barbershop world) is really a moment for the baritone to be melodic and distinct from the quartet, in contrast to his usual goal of being blended and unnoticed. The notated pitches are effective at bringing attention to the baritone without disrupting the overall progression. The third beat goes by quickly and few listeners will pay attention to the harmonic blend. Though I've thoroughly described the issue, obsessing on the various ratio options in every case seems misguided. Sometimes melody really is the focus and that is all that matters.

Measure 12 contains a ninth chord that includes the root and so is unambiguous, in contrast to half-diminished chords that were described earlier as ninths-without-roots. This rooted ninth is a particularly rich, ringing chord, especially with the optional low bass which better supports the ninth. The chord on beat two is an embellishment, a relative minor to the V chord, and so will be tuned 12:15:20:24. This tuning is the only reasonable choice because the overriding harmony of the measure is clearly the dominant V chord, and the tenor is resolving into the root from the ninth of the previous chord. If the lead did not move and the bass moved to the fifth of the chord, it would result in another Chinese seventh version of a barbershop seventh chord, which might be more typical in this situation. As written, the lead moves to harmonize at a sixth, and this chord should be called E♭6 instead of Cm, similar to the situation in m. 2. Then, the return to the V chord varies only in being a barbershop seventh instead of a ninth.

On “one more,” the tenor takes over the most prominent melody. Actually, the melodic focus changes initially in m. 11 when the prominent baritone takes focus away from the lead. At that point, there is no clarity whether the main melody is in the lead or tenor. On the downbeat of m.12, the strong harmonic unity creates a cohesive blend that de-emphasizes any one voice. The tenor melody comes out on the leading-tone at “more,” but the tenor melody is only fully clear when considered retrospectively upon hearing the final tonic note in m. 13.

Measures 13-16: The tag reharmonizes the melody post until final resolve

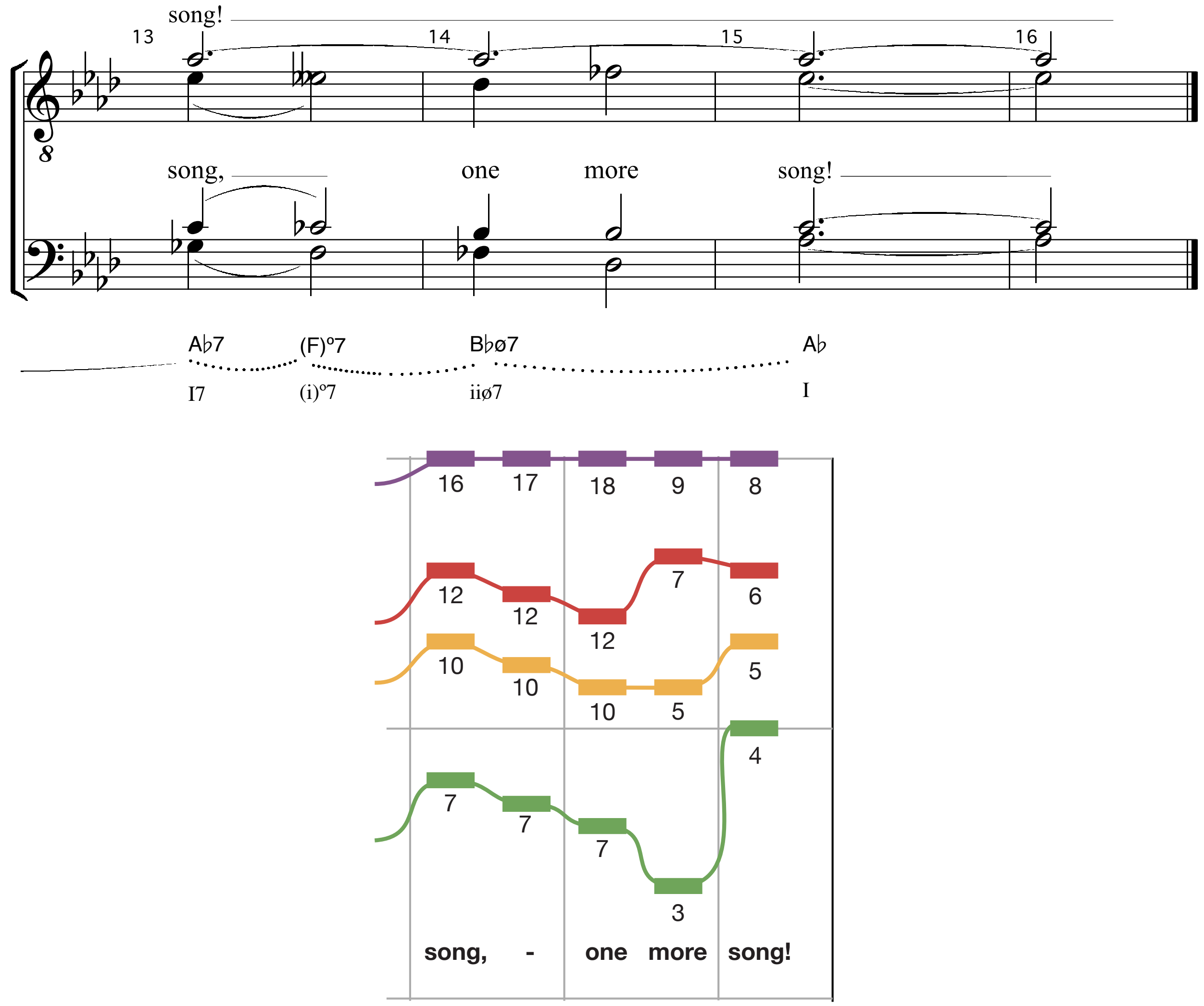

Figure 14. The tag, m. 13 to the end.

In barbershop, after the final note of the song's melody arrives, a *tag* adds extra embellishments before the final harmonic resolve. The term is used primarily for songs not originally in barbershop style when new barbershop-specific musical content is tagged onto the end after the basic song is completed. In barbershop originals, such as *One More Song*, similar end content is typical, and the term *tag* still makes sense. Although not the only possibility, it is common for tags to have the melody note sustain (*post* in barbershop terms) while the other parts harmonize around it, as occurs here.

In the *One More Song* tag, initial resolution is avoided by putting the bass on the seventh. Notice that this inversion with the 7 in the bass creates a much higher combined ratio than in all the other barbershop seventh chords in the rest of the song. Then, all three harmony parts dip, creating a fully-diminished chord that simply reharmonizes the tenor from 16 to 17. In this remarkable case, the diminished chord has a blend comparable to the inverted barbershop seventh on beat one. To clarify, the barbershop seventh inversion here has one of the highest possible ratios for a type of chord that is usually quite blended; and the fully-diminished here is nearly the lowest possible ratio for a chord-type that is one of the roughest in barbershop. The three lower parts then continue the pattern and drop down another half-step, putting the tenor at 18. This same chord is then re-voiced by swapping the lead and bass, thus putting the 7 in a higher octave, resulting in a lower — essentially ideal — ratio of 3:5:7:9. As stated earlier, it is harmonically a ♭VII9 with no root. In other words, it tunes as a barbershop-seventh one whole-tone below the tonic post, thus making the post into a ninth. This chord is very common in barbershop tags. The song then resolves in m. 15 to the final tonic, which matches the initial tag chord only with the bass now finally on the root.

If the baritone were to jump up an octave on the last note (the best arrangement for smooth transition would put the baritone up an octave on both of the last two notes), then the final chord would be an even simpler 2:3:4:5 ratio. Such a rearrangement would result in a fundamental an entire octave higher than any fundamental in the rest of the piece (fundamentals are charted in the full graph Appendix C). Although the baritone singing above the tenor is unusual and the pitch is unusually high, I have witnessed a number of occasions in which adventuresome singers or arrangers actually make this exact modification in just this type of context. The final chord in the arrangement has a fundamental in a high enough range that it blends quite well as written and better than the same structure an octave lower; but the willingness of singers to stretch their voice range to achieve the even lower ratio is further evidence of the influence of harmonic ratio on choices in barbershop.

The importance of absolute pitch

Absolute pitch is an issue that warrants further discussion. Beats, listener sensitivity, and critical bands all vary with absolute pitch. As readers may have noticed in the analysis, when any odd-numbered pitch in a ratio is moved down an octave, it necessarily causes a doubling in the ratio numerals of the other voices. However, any pitch may be moved up an octave and the result will either be a lower overall ratio or have no impact on the numerals in other parts. This is one reason that there is more tuning flexibility for music in higher pitch ranges. The analysis here is particular to male vocal range. The issues of blend and various ratios is somewhat different for all-female barbershop quartets.[35] Because they sing in higher ranges, female harmonies have less

[35] sometimes called “beauty-shop” quartets or “Sweet Adelines,” a reference to the song and to the female barbershop society.

potential to create roughness. This allows female singers to have more varied, higher-ratio, or more readily tempered harmonies with less roughness.[36] In mixed vocal groups, the ratios in the lower men's ranges are constrained in the same way as all-male quartets and the higher female voices will ideally fit into the same harmonic series as the lower voices. Of course, because there are far more harmonics in the upper range of the harmonic series, upper voices still have much more flexibility than lower voices, which is one reason that bass lines have more typically jumpy contour than other voices: the bass range has fewer and further apart harmonics from which to choose if chords are to blend well (given relatively stable overall harmonies in which to fit).

In the full ratio graph for *One More Song* (Appendix C), I have marked the missing fundamental of each chord. Notice that the highest fundamental is a tonic note one octave below the bass singer's lowest tonic.[37] This highest fundamental occurs at four spots, all of which are tonic chords and are the strongest resolutions in the song. The rest of the chords have fundamentals mostly ranging within the two octaves below the highest fundamental. In this key, the lower fundamentals in the arrangement are in infrasonic (below 20Hz) range.

I think it is helpful to view the Appendix C chart while considering this metaphor: imagine the 52Hz A♭ to be the surface of a body of water, upon which a ball is floating. The pressure of each harmony pushes the ball into the water. The ball's depth corresponds to varying degrees of tension that release when the pressure is removed and the ball returns to the surface. This seems to me to correlate somewhat with the experience that I have listening to the song and

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[36] Fewer mid-range partials in the spectrum of women's voices also means that they have more limited options for bringing out partials by adjusting formants.

[37] Although the optional modification of the final chord (mentioned in the discussion of the tag) would offer a higher fundamental that would match the lowest bass pitch.

the blend of the chords. Of course, the degrees of beating, tension, and other issues are affected by complexities of part balance and timbre, etc.

Concluding thoughts

The computer-adjusted recordings accompanying this paper demonstrate audible differences in various tunings and the effects of achieving near harmonic ratios. But when pitches are off from a ratio only slightly, the result is simply mild beating which adds some energy and interest to the harmony but doesn't harm the overall effect. Furthermore, regular beating is diminished by the inherent inconsistencies of live voices. A bit too far from the Just ratio, however, roughness can still become more noticeable, perhaps obscuring the harmonic blend. With four singers, there are nearly countless possibly interacting tones within the complex combination of all the partials of all the voices along with all the combination tones. Tunings beyond modest deviation from harmonic ratios may become more cacophonous instead of a rich, unified sound. Tuning tolerance varies based on the particular ratio, the absolute pitch, vocal timbres, part balance, and many other factors; and all the parameters must be weighed against the broader purpose of the music, i.e. how these choices relate to expressive musical intentions.

Overall, harmonic ratio theory can greatly inform the study of barbershop style. The ratio analysis provides insight into the arrangement where the standard notation is opaque. Ratios do appear to correlate well with the arrangement (and actually correlate even better with less ambiguity in many other arrangements I have analyzed), and this is evidence for the idea that ratios are a foundation for barbershop harmony despite the many complexities in the realization of the music by live voices.

The final significance of any music theory, of course, is in its applicability to the experience of the people involved. I hope this presentation provides value to singers and arrangers and adequately sets the stage for future empirical testing of the many hypotheses mentioned.

The barbershop singer's experience

Considering all the issues involved, any particular barbershop singer is: listening to the blend of the group harmony, hearing their own voice's place in each chord, following their own voice's melodic motion (even in harmony parts, but especially in the lead), controlling their vocal quality and physical support, creating effective rhythm (particularly in synchronizing with the other singers), expressing the meaning of the lyrics, and engaging in the emotional flow of the song. Public performance adds further presentation elements. If singers satisfy their goals on all these counts, they can consider themselves successful. Great singers develop effective habits for the majority of their concerns and then during performance will pay conscious attention to only a few or possibly none of them. When first learning how to deal with each of these issues, singers may be more or less cognitive and intentional. Indeed, any given singer has a degree of familiarity with the style, ability to read notation or not, and various explicit ideas from coaches and other sources. It is reasonable to assume that no quartet will ever achieve near-perfection on all of these independent goals. Holistically, we should note that the separate goals impact one another, and yet particular effectiveness in one area can also mask detriments in others. Many questions for further study involve potential correlations between these factors.

If singers are exposed to the sound of controlled, precise harmonic ratios (and of various deviations from precise ratios), they will be more sensitive to the possibilities than if they only

hear the complexities of live singing or the tempered tuning of the piano. If singers become aware of the issues relating melody and harmony, they will be more aware of their interpretive options rather than believing that there is one objectively correct tuning. Understanding harmonic ratios is also obviously useful in creating computer-adjusted recordings, which are already widespread in the evolution of barbershop music into the 21st Century.

Because a barbershop arrangement scored in standard notation can be effectively subjected to harmonic ratio analysis, we must accept that standard notation, as problematic as it may be, is a capable vehicle for barbershop harmony. Nevertheless, novel notations may be helpful in pedagogy (perhaps, for example, influenced by my pitch graph; or perhaps using novel accidentals, or tuning indications via note-head shapes, among other possibilities). Of course, some deference should still be given to implicit arranging via woodshedding, and theorists should be sensitive to the problems of imposing their theories on the music.

Harmonic-ratio-informed arranging choices

Though harmonic ratio approximation may already have implicit impact in determination of barbershop arranging choices, explicit understanding can greatly inform the arranging process. *One More Song* is a particularly problematic arrangement for tuning analysis, but not all arrangements are necessarily comparable. While using the same palette of chord-types and the same general syntax, an arranger with knowledge of ratio theory may be able to avoid tuning ambiguity and also more easily discover embellishment possibilities. When a chord progression requires a non-obvious tuning adjustment, an informed arranger can swap voices or add transitional movements to mask shifts or avoid the problem entirely. When an arranger wants to add tension or assist further blend, harmonic ratio awareness makes the decisions much more

obvious. For novice arrangers, conceiving of the music by harmonic ratios can be a more concrete and accessible approach that greatly clarifies the complex and seemingly-arbitrary traditional stylistic rules. Even considering all the complexities discussed in this treatise, and even when singers do not match theoretical tuning, the same arrangements that are less ambiguous in harmonic ratio analysis are also the most accessible to singers.[38]

How the harmonic ratio theory relates to other music styles

The lessons from this analysis and discussion of barbershop can be extended to explain why certain chords and inversions are chosen in other music styles. For example, folk and pop guitarists prefer certain chord shapes not because they are too lazy or untrained to understand chord inversion theory (a claim I've heard from biased guitarists with Western theory training). Instead, the most common chords in folk and pop guitar are generally those where the tempered tuning is closest to Just ratios (as in *power chords* which are open perfect fifths, and suspended fourths and ninths) and those with structures that best approximate low-number ratios.[39] There are similar correlations between the harmonic series and the implicit choices made by folk musicians in many cultures.[40] These choices are emphasized and highlighted in barbershop harmony, with its extraordinary emphasis on blended chords.

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[38] For examples of particularly unambiguous arrangements, see the *Heritage of Harmony Songbook* (Szabo, 1988), which is now available for free at barbershop.org. In particular, I have analyzed *The Sidewalks of New York* which is built entirely on I, IV, V7, II7, and VI7 chords with 6th and 9th embellishments and virtually no occasions of ambiguous pitch shifts, drifts, or any other problems. It is a notably easy and accessible arrangement as well. Significantly, Szabo contends that his arrangements (and song choices) are more in line with the primary roots of barbershop style in contrast with many newer arrangements and songs.

[39] This conclusion is based on my many years of studying and teaching guitar, including thorough reviews (currently unpublished) of more than seven hundred guitar methods, songbooks, and related publications.

[40] While much of this theory applies beyond barbershop, the specific ratios of Just Intonation are only substantially relevant to harmonic timbres. Blend and tuning play out differently in music based on inharmonic bells, gongs, bars, and drums. Even plucked or hammered strings (e.g. guitar or piano) have spectra that are slightly inharmonic, deviating in subtle ways from a perfect harmonic series. Of course nearly all music cultures use at least one harmonic sound: the human voice. Regardless of harmonic or inharmonic spectra, the general concept of matching partials may be relevant to all music (Sethares, 2005).

References

Averill, G. (2003). *Four Parts, No Waiting: A Social History of American Barbershop Quartet.* New York: Oxford University Press.

Barbershop Arranging Manual. (1980). Kenosha, WI: SPEBSQSA, Inc.

Devaney, J. & Ellis, D. P. W. (2008). An empirical approach to studying intonation tendencies in polyphonic vocal performances. *Journal of Interdisciplinary Music Studies, 2(1-2)*: 141-156.

Eskelin, G. (2005). *Components of Vocal Blend, Plus: Expressive Tuning.* Woodland Hills, CA: Stage 3 Publishing.

Hagerman, B. & Sundberg, J. (1980). Fundamental Frequency Adjustment in Barbershop Singing, *Journal of Research in Singing 4/1*: 3–17.

Hahn, K. & Vitouch, O. (2002). Preference for musical tuning systems: How cognitive anatomy interacts with cultural shape. In C. Stephens, D. Burnham, G. McPherson, E. Schubert & J. Renwick (Eds.), *Proceedings of the 7th international Conference on Music Perception and Cognition* (pp. 757-860). Adelaide: Causal Productions.

Hallam S., Cross, I. & Thaut, M., (Eds.) (2009). *The Oxford Handbook of Music Psychology*. Oxford: Oxford University Press.

Helmholtz, H. von. (1885). *On the sensations of tone as a physiological basis for the theory of music* (4th ed.) (Ellis, A. J., Trans. plus notes). Reprinted New York: Dover, 1954.

Kalin, G. (2005). *Formant frequency adjustment in barbershop quartet singing* (Master's thesis). Dept. Speech Music and Hearing, Kungliga Tekniska Högskolan, Stockholm, Sweden

Liles, J. (1985). *One More Song*. Kenosha, WI: SPEBSQSA, Inc., New edition (1999) available online <http://barbershop.org/document-center/category/19-free-music-for-printing.html>

Loosen, F. (1995). The Effect of Musical Experience on the Conception of Accurate Tuning. *Music Perception, 12(3)*: 291-306

Nordmark, J., & Ternstrom, S. 1996. Intonation preferences for major thirds with non-beating ensemble sounds. *TMH-QPSR Speech, Music and Hearing: Quarterly Progress and Status Report*. 1, 57-61

Richards, J. (2001). *The Physics of Barbershop Sound.* Kenosha, WI: SPEBSQSA, Inc.

Sethares, W. A. (2005). *Tuning, Timbre, Spectrum, Scale* (2nd ed.). London: Springer-Verlag.

Smucker, P. (2010, April). *Interpretive Timing in Barbershop Quartets*. Unpublished paper presented at the Annual Meeting of the Midwest Chapter of the Society for Ethnomusicology, Chicago.

Snyder, B. (2000). *Music and Memory: An Introduction.* Cambridge, MA: The MIT Press.

Szabo, B. (1976). *Theory of Barbershop Harmony.* Kenosha, WI: SPEBSQSA, Inc.

Szabo, B. (1988). *Heritage of Harmony Songbook.* Kenosha, WI: SPEBSQSA, Inc. Available in print but now also online for free at <http://barbershop.org/document-center/category/20-heritage-of-harmony-sheet-music.html>

Wild, J. & Schubert, P. (2008) Historically Informed Retuning of Polyphonic Vocal Performance. *Journal of Interdisciplinary Music Studies 2(1-2)*: 121-139

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Appendix A: Accompanying audio files list and discussion

The recordings accompanying this treatise are listed below. While there are an infinite number of parameters that could potentially be highlighted in different recordings of just one piece of music, the included recordings highlight some of the issues most emphasized in the analysis, particularly harmonic tuning.

Table 1. *Accompanying audio files*[41]

track /file	**section**	**voices**	**tuning**	**computerized pitch correction**	**tempo**
1	complete	full quartet	unedited (but I practiced along with Just Intonation tuning)	unedited	normal
2	complete	choir made from combining all the takes from my recording session	unedited	unedited	normal
3	complete	full quartet	Just Intonation (primary choices as shown in the full graph in Appendix C)	only slight correction, only very minor vibrato reduction	normal
4	complete	full quartet	Just Intonation	medium correction	normal
5	complete	full quartet	Just Intonation	near-complete correction, substantial vibrato reduction, portamento still	normal
6	complete	full quartet	Just Intonation	complete correction, no portamento, no vibrato	normal
7	complete	full quartet	Equal Temperament	near-complete	normal

[41] The term correction in the table is problematic because it implies a proscriptive judgment. No broad implication is intended. The value of any tuning or adjustment may only be judged in reference to its purpose. In this case, these recordings are correct to the extent that they successfully demonstrate each tuning issue as intended.

track /file	section	voices	tuning	computerized pitch correction	tempo
8	complete	full quartet	Just Intonation	near-complete	quarter-speed
9	phrase one mm. 1-4	full quartet	Equal Temperament	near-complete	quarter-speed
10	complete	tenor part track (selected part panned right, others panned left)	Just Intonation	near-complete	normal
11	complete	lead part track	Just Intonation	near-complete	normal
12	complete	baritone part track	Just Intonation	near-complete	normal
13	complete	bass part track	Just Intonation	near-complete	normal
14	pickup into first downbeat of m. 1	full quartet	comparison baritone flattening to seventh, sustaining tonic, or others tuning sharp	near-complete	half-speed
15	end of m. 1 into m. 2, passing swipe	full quartet	comparison of multiple tunings	near-complete	half-speed
16	m. 3 to downbeat m. 4	full quartet	comparison of III7 tuned to tonic's third or one comma sharp	near-complete	normal
17	m. 7	full quartet and lead isolated stereo	comparison of tunings for beat 2	near-complete	half-speed
18	pickup into downbeat of m. 9	full quartet	comparison of various augmented tunings	near-complete	quarter-speed

I created all the audio files from an initial recording in which I overdubbed each voice part. Without the holistic experience of singing in a well-rehearsed live quartet, the tuning and match of each part was not ideal. I created an initial version with some editing and practiced with that before recording the final tracks. Audio File 1 is the unedited mix of the final selected takes for each part. The result is reasonably musical and fluctuates between more or less well-tuned harmonies, none of which are fully consistent throughout their duration.

Audio File 2 is a novel mix of all my recorded takes played back at the same time along with mild artificial reverberation and no further editing. My overall interpretation was consistent but my pitch, dynamics, and vocal quality varied slightly from take to take. The resulting chorus sounds remarkably like a large barbershop chorus or perhaps like a “gang-sing” as heard at a barbershop convention. The subtle pitch inconsistencies from Audio File 1 are completely masked in Audio File 2. The chorus effect creates a complexity that I find opaque in terms of precise tuning. Blend, tonality, and overall musical clarity are strong.

Audio Files 3-6 demonstrate the effect of computerized pitch adjustment. As the pitch becomes more precisely controlled and matched to exact harmonic ratios (to one-cent accuracy at maximum in audio File 6), the cohesion and blend in the chords becomes stronger. The sense of natural singing is compromised, however, as the pure tunings create an intense (but unrealistic) buzz and match. Also, as the harmony blends further, other nuances are more apparent. My personal feeling is that of altered sense of time. As I become more aware of instantaneous subtle details, I also hear long chords more cohesively as singular events. Beyond medium correction definitely sounds less like any live quartet, but is effective here for demonstrating the theoretical tuning issues.

Audio File 7 demonstrates equal tempered tuning. The result is still undeniably musical, with the same overall content and still perceptible harmony. However, beats are now apparent, more in some chords than in others. The previous subtlety and flexibility of pitch is gone as everything fits into a simpler, more rigid set. The tempered tuning causes more constant tension in the sound. Compared with the Just Intonation version which ranges from very blended and strong to more rough and complex, the tempered recording has much less affective range.

Nevertheless, it is notable that the same chords are more or less blended in both equal tempered and Just Intonation versions.

Audio File 8 demonstrates the Just Intonation tuning at quarter speed. I included this to demonstrate with particular clarity the exact tuning of each chord. Audio File 9 contains one phrase of quarter-speed Equal Temperament so that listeners can more readily compare the Tempered and Just tunings at this slow speed. I believe one phrase is enough to sense the effect Tempered tuning.

Audio Files 10-13 are mixed as part tracks, in the same format typically used to teach barbershop songs. By using a balance (pan left or right) control during playback, listeners may choose to hear only the one voice in question, or only the other three. This is especially useful in studying the subtle melodic effects discussed in the analysis.

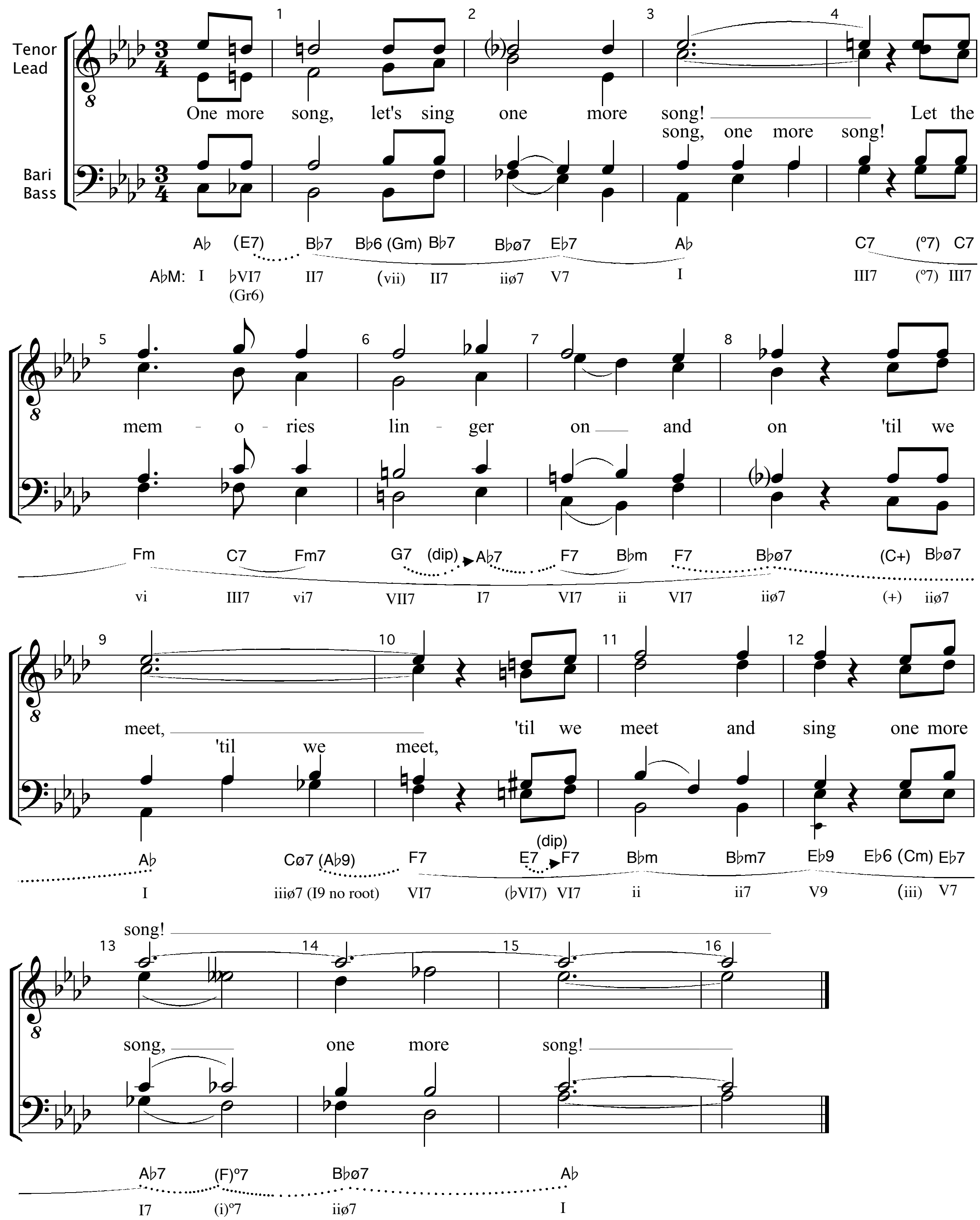
Audio Files 14-18 demonstrate various alternative tunings for particular spots discussed in the analysis. Narrated introductions identify each example. Use of more advanced playback software will allow easy access to specific spots within each audio file for the purposes of repetition and further comparison. It is very hard to notice these subtle differences with only limited initial exposure.

It was challenging to decide whether to slow down these examples further. When heard at faster tempos, the subtleties are hard to perceive. At slow tempos, however, it becomes harder to compare the differences entirely within short-term memory. Especially slow tempos are also, of course, less representative of normal performance. My intention is to show the general types of issues involved in the style rather than to over-emphasize particular tuning situations.

If someone has trouble perceiving any difference in certain distinction examples in the audio, then the distinctions are, for that person, irrelevant. Indeed, these subtle distinctions are not presented to suggest that they necessarily have major consequence. I perceive all the distinctions clearly after the substantial reinforcement of hearing them during the process of creating the files, not to mention my years of past study of similar issue. Nevertheless, I still sometimes feel that some distinctions are subtle and inconsequential.

All sorts of additional variations could have been added, of course, if time and practicality allowed. A recording that alternates between Equal Temperament and Just Intonation could be of interest. I suspect that if Just Intonation were used for the most resolved chords and Equal Temperament for the others, it would be very effective because tension and resolve would remain consistent with the normal song structure and perhaps even be enhanced. I think the included recordings show that insistence on choosing one best ratio for every chord in an arrangement is not always justified, yet also show that there remains some perceptible significance to issues of precise tuning. As emphasized throughout this paper, all of this brings up numerous questions and there is need for a wide variety of future empirical research.

Appendix B: *One More Song* score analysis



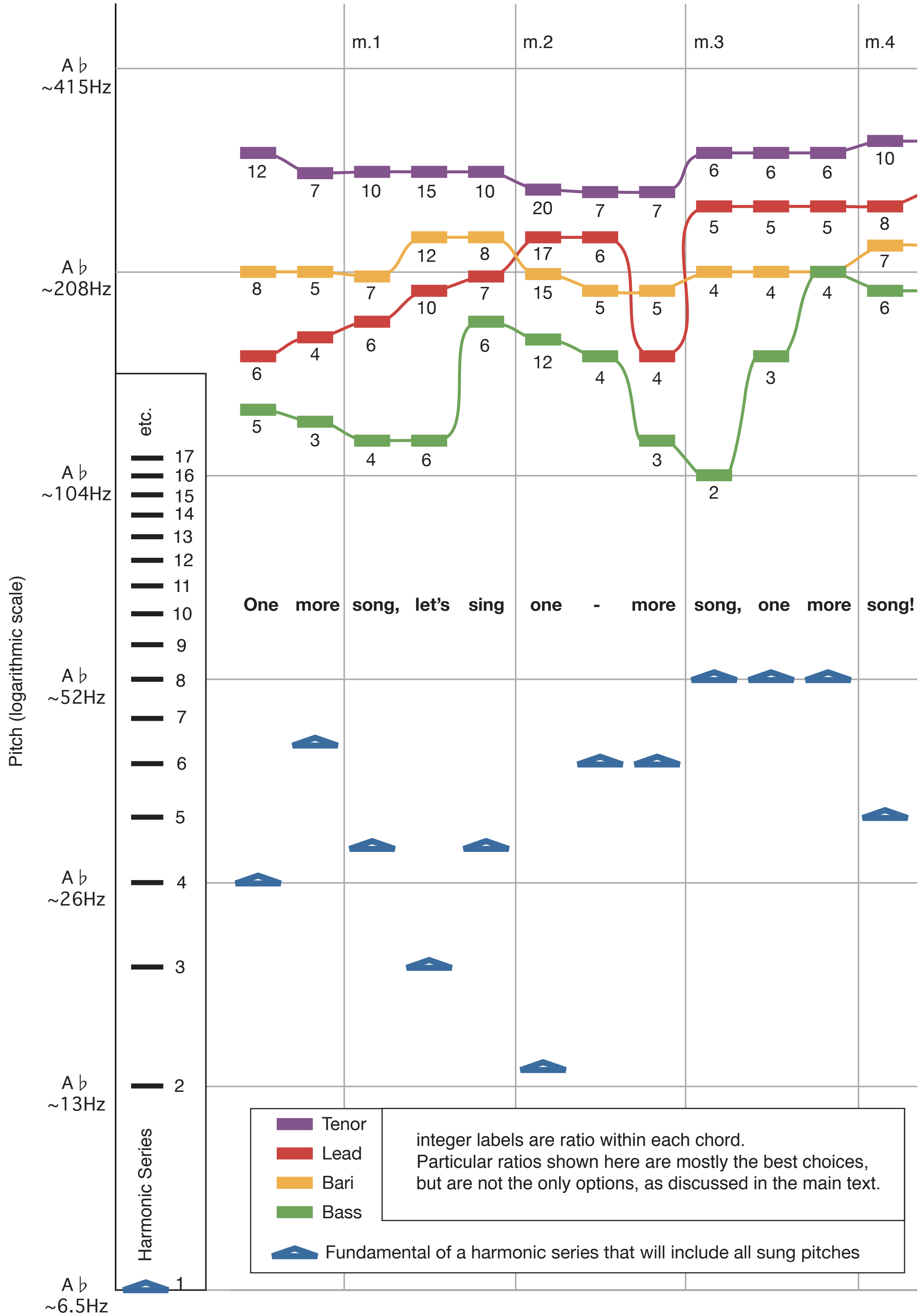
Further notes on macro analysis

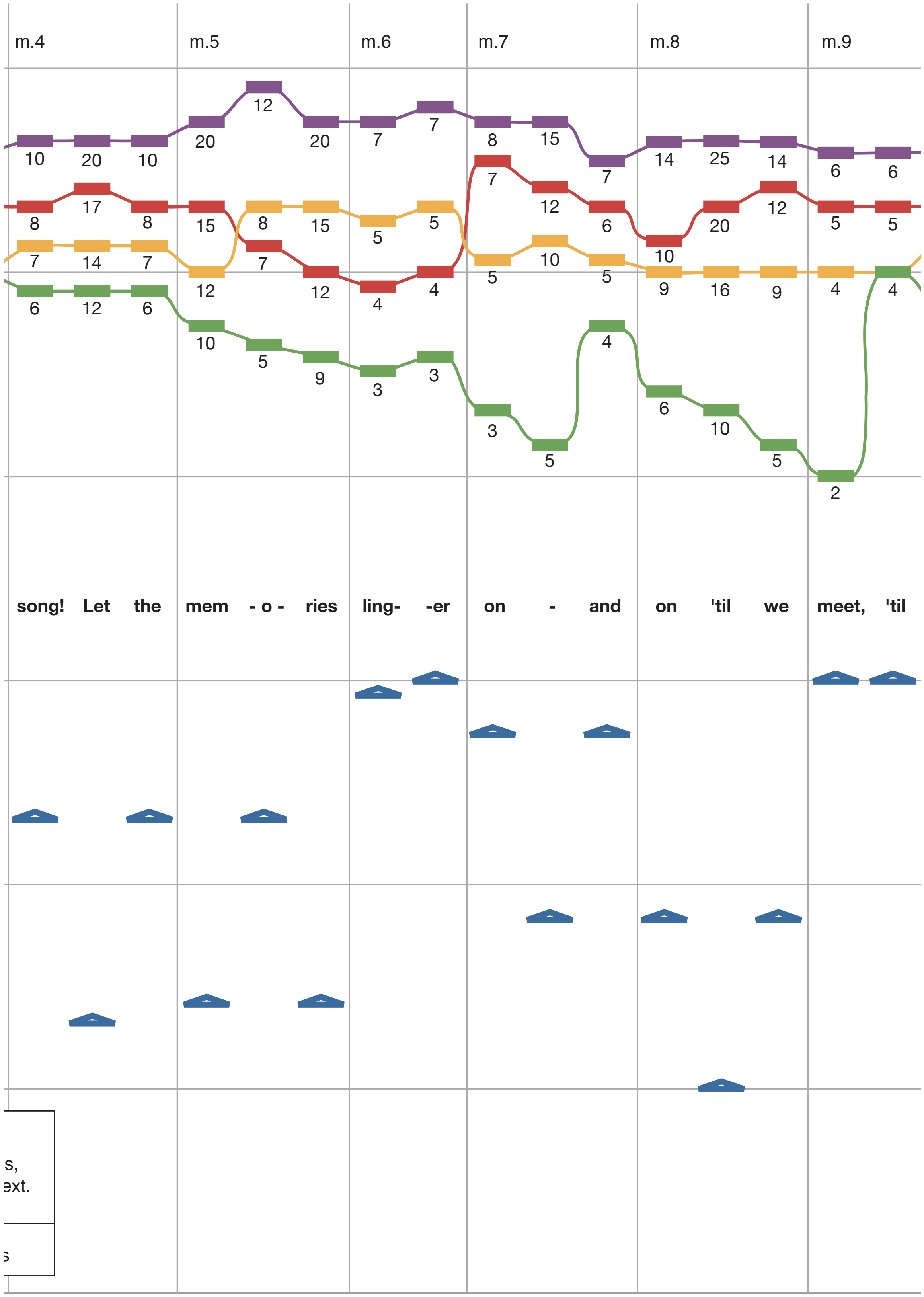
This macro analysis indicates harmonic progressions and traditional chord names. In balancing classical versus popular/jazz names, I've chosen to ignore inversions, which are apparent anyway in the notation and in the ratio analysis. Figured-bass inversion symbols are not used in most barbershop contexts, do not easily allow analysis of ideas such as chords with missing roots, and create unnecessary complexity for the purpose here. Also, secondary dominants here are written based on their scale degree, (e.g. II7 instead of V7/V), as commonly done in barbershop, jazz, and popular terminology. As discussed in the paper, some of these chords have possible alternative names, and, in choosing among the options, I tried to balance between the most common choices with those that best relate to the particular context.

Solid slurs indicate dominant (descending fifth) circle progressions. Dotted slurs indicate alternative-yet-also-strong leading progressions including: tritone substitution (which is generally really a semitone-above chord simply leading downward), subtonic 9th chords resolving up to tonic (or functional substitution), and common alterations of circle progressions (e.g. leading-tone chords).

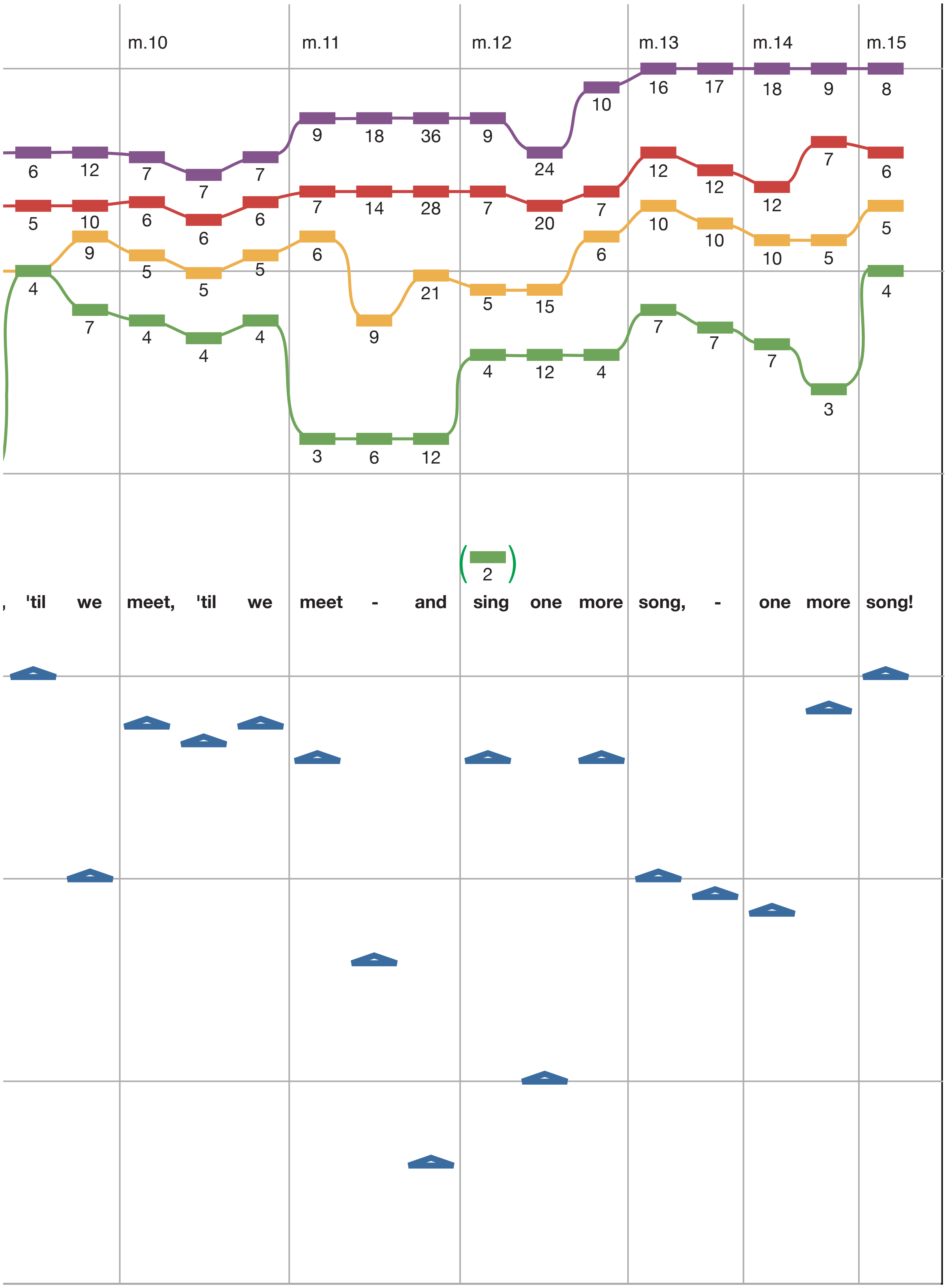
Naming issues of more ambiguous chords (such as diminished and augmented) are discussed along with many other issues in the main text. In particular: ø7 = *half-diminished seven* in classical terms, a.k.a. m7♭5 in jazz/popular terms, which is also the same notes as *minor-add-6* (e.g. B♭ø7 = D♭m6). None of those names provide clarity regarding tuning. However, when this type of chord is tuned with certain ratios (given in close-position), 5:6:7:9 justifies calling the chord a *dominant-ninth-no-root* (e.g. B♭ø7 ≈ G♭9-no-root); and 10:12:15:17 could be called a *major-seven-flat-nine-no-root* (e.g. B♭ø7 ≈ B♭♭M7♭9-no-root).

Appendix C: Pitch Ratio Graph





m.4
m.5
m.6
m.7
m.8
m.9
10 20 10 20 12 20 7 7 8 15 7 14 25 14 6 6
8 17 8 15 7 12 4 4 7 12 6 10 20 12 5 5
7 14 7 12 8 15 5 5 5 10 5 9 16 9 4 4
6 12 6 10 5 9 3 3 3 5 4 6 10 5 2 4
song! Let the mem - o - ries ling- -er on - and on 'til we meet, 'til


m.10
m.11
m.12
m.13
m.14
m.15
, 'til we meet, 'til we meet - and sing one more song, - one more song!

Further notes on the graph

Like standard Western notation, the graph shows pitch in the vertical dimension and time in the horizontal. Unlike notation, the graph does not favor any particular scale construction. Instead, the graph shows pitch on a logarithmic scale without any strict categories. Also, time is not shown analogously; rather, the graph only shows sequence order. The bold rectangles represent the concept of one note, and I intentionally made them thick to imply a subtle degree of categorical flexibility. However, the exact placement of each mark is precise to the tuning shown. Curved connecting lines indicate the smooth flow of contour and are not meant to be precise; they imply moments of transition between notes. These transitions may be realized in various ways by any singer and are not the focus of the present analysis. Nevertheless, I still made an attempt to have mostly horizontal lines for repeated pitches and angled lines when pitches shift slightly. I could have eliminated the connecting lines during rests or even short breaths, but I felt that wasn’t necessary. The full graph and the bits shown in figures alongside notation are the primary tuning choices I selected, and these are demonstrated in the full length audio files. Some alternatives are shown in additional figures in the analysis, while other options are discussed in the text with no illustration.

Appendix D: Translation Chart of Scale Names and Harmonic Series

Full explanation is beyond the scope of this paper; but in relating the lines, imagine them continuing indefinitely, and some being independently movable for alternate alignments.